MA 135: College Algebra (Fall 2016)
Instructor: Justin Ryan
Week 2 Notes

How to use this handout—This handout contains a skeleton of the notes that we will study in class this week. I've typed out definitions and theorems so that you don't have to exasperatedly copy what I'm writing, and populated these pages with a number of examples. My expectation of you is that you will fill in all of the details, ideas, *etc*, that I've left out.

Section 1.5—Linear Equations and Inequalities

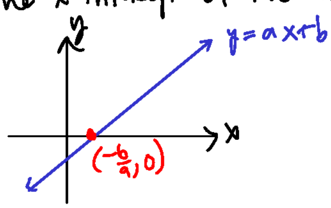
Linear Equations

A linear equation can be written in the form

$$ax+b=0$$

Solutions of Linear Equations

The solution of a linear equation, written in the above form, is $x=-\frac{b}{a}$. This number represents the x-value of the x-intercept of the line.



"Cancellation" Properties

Let a, b, c represent real numbers. Then

1.) $a+c=b+c$ implies that $a=b$.

and

2.) $a\cdot c=b\cdot c$ implies that $a=b$.

Example Solve for x: $10+3(2x-4)=17-1(x+5)$

$$10+6x-12=17-x-5$$

$$6x-2=-x+12$$

$$6x+x=2+12$$

$$7x=14$$

$$\boxed{x=2}$$

Example Solve for x: $\dfrac{x+7}{6}+\dfrac{2x-8}{2}=-4$

$$6\left(\frac{x+7}{6}+\frac{2x-8}{2}=-4\right)$$

$$\cancel{6}\cdot\frac{x+7}{\cancel{6}}+\overset{3}{\cancel{6}}\cdot\frac{2x-8}{\cancel{2}}=6(-4)$$

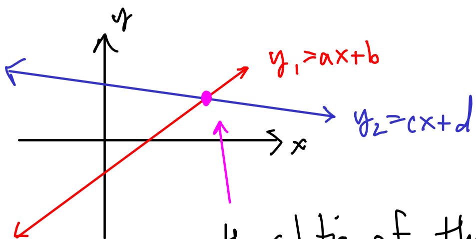
$$x+7+3(2x-8)=-24$$

$$x+7+6x-\cancel{24}=-\cancel{24}\quad 0$$

$$7x+7=0 \rightarrow 7x=-7 \rightarrow \boxed{x=-1}$$

Graphical Approach

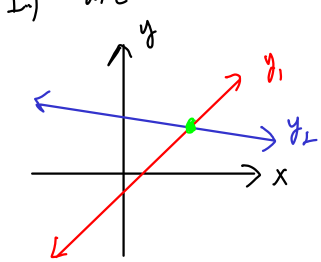
Consider $\underbrace{ax+b}_{y_1}=\underbrace{cx+d}_{y_2}$



the solution of the equation is the x-value of the point where the lines intersect. The y-value is not explicitly part of the soln.

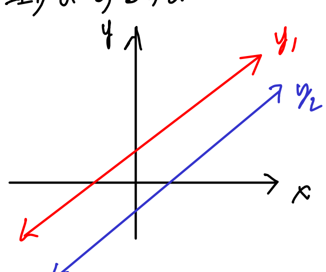
Three Possibilities:

I) $a\neq c$



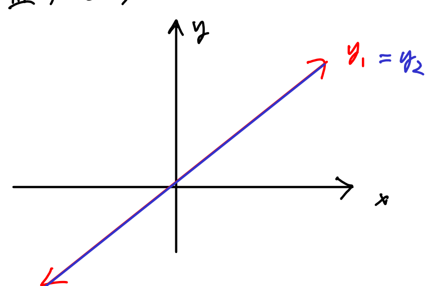
ONE solution

II) $a=c,\ b\neq d$



NO solutions

III.) $a=c,\ b=d$

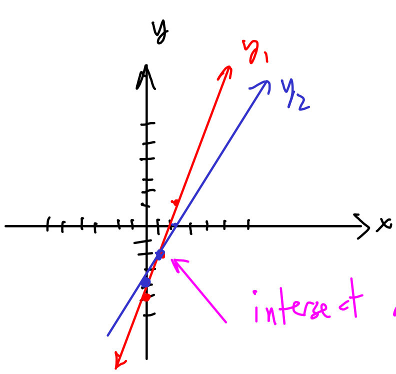


∞-many solutions: All real numbers, $\mathbb{R}$

Example Solve for x: $3x-5=2(x-2)$

$y_1=3x-5$

$y_2=2x-4$



intersect at $(1,-2)$, so the solution is $\boxed{x=1}$

Linear Inequalities

A linear inequality can be written in one of the forms:

$$\begin{cases} ax+b<0 \\ ax+b\leq 0 \\ ax+b>0 \\ ax+b\geq 0 \end{cases}$$

"Cancellation" Properties

Let a,b,c be real numbers. Then

1.) $a+c<b+c$ implies $a<b$;

2.) if c is positive, then

$a\cdot c<b\cdot c$ implies $a<b$;

3.) if c is negative, then

$a\cdot c<b\cdot c$ implies $a>b$. !

Example Solve for x and plot the solution on a number line.

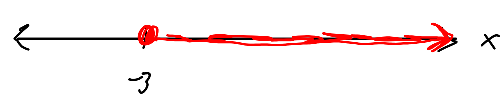
$$3x-2(2x+4)\leq 2x+1$$

$$3x-4x-8\leq 2x+1$$

$$-x-8\leq 2x+1$$

$$-9\leq 3x$$

$$\boxed{x\geq -3}$$



Example Solve the inequalities for x and plot the solution on a number line.

(*a*) $2x-3 < \dfrac{x+2}{-3}$

$$-3(2x-3) > x+2$$
$$-6x+9 > x+2$$
$$7 > 7x$$
$$\boxed{x<1}$$

(*b*) $-3(4x-4) \geq 4-(x-1)$

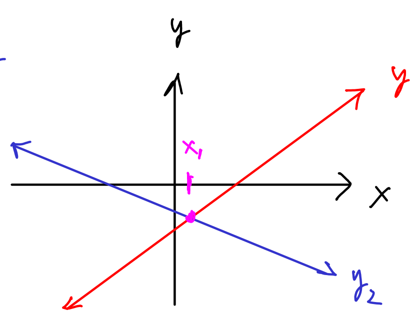
$$-12x+12 \geq 4-x+1$$
$$-12x+12 \geq -x+5$$
$$7 \geq 11x$$
$$\boxed{x \leq 7/11}$$

Graphical Approach

$$\underbrace{ax+b}_{y_1} \leq \underbrace{cx+d}_{y_2}$$

$y_1 = ax+b$

$y_2 = cx+d$



The height of the graph of y_1 is less than or equal to that of y_2 for $\boxed{x \leq x_1}$

Example Solve the inequality for x: $\underbrace{3x-2(2x+4)}_{y_1} \leq \underbrace{2x+1}_{y_2}$

$y_1 = 3x-4x-8$

$y_1 = -x-8$

$y_2 = 2x+1$

The graphs intersect at $(-3,-5)$. The graph of y_1 is "lower" for all x-values greater that -3.

Sol'n: $\boxed{x \geq -3}$