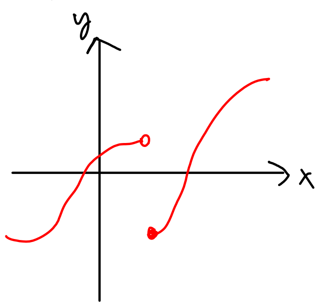
Definition A function is **continuous** on an interval of its domain if and only if

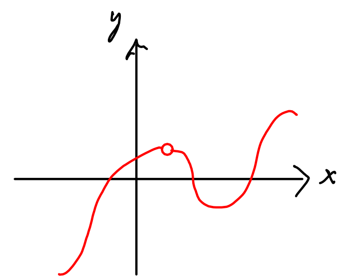
the graph can be drawn without lifting our pencil.

Examples of discontinuities

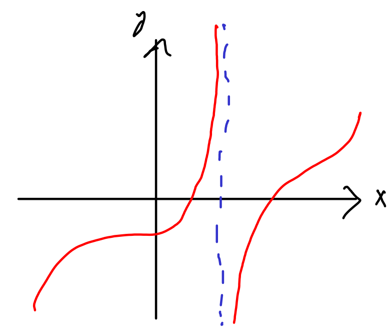
jumps, holes, and asymptotes.



JUMP



HOLE



ASYMPTOTE

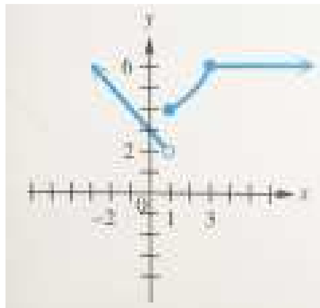
holes are "removable". We can "fill them in" by defining the function to equal what it's "supposed to" at that x-value.

Increasing, Decreasing, and Constant functions

A function is $\left\{\begin{matrix}\text{increasing}\\ \text{decreasing}\\ \text{constant}\end{matrix}\right\}$ on an interval (a,b) if and only if

$$\left\{\begin{matrix} f(c_1)<f(c_2) \\ f(c_1)>f(c_2) \\ f(c_1)=f(c_2)\end{matrix}\right\} \text{ for all } c_1<c_2 \text{ such that } a<c_1<c_2<b.$$

Example Determine the intervals on which the function is increasing, decreasing, and/or constant.



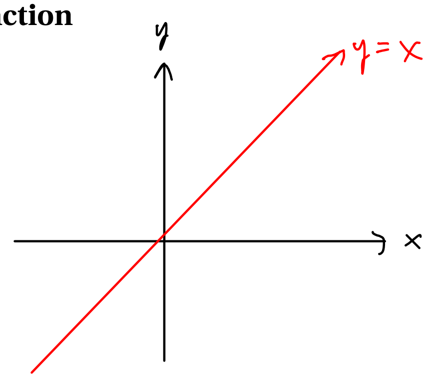
inc: $(1,3)$

dec: $(-\infty,1)$

const: $(3,\infty)$

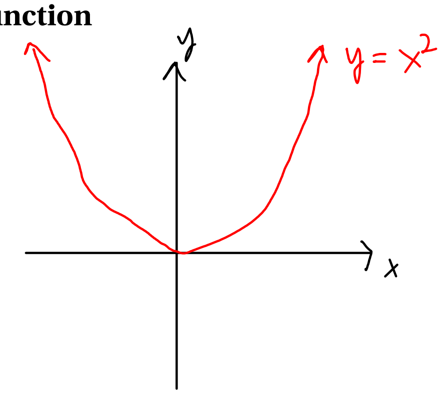
The Identity Function

$f(x) = x$



The Squaring Function

$f(x) = x^2$



Symmetries of the Identity and Squaring Functions

The identity function has symmetry about the origin (rotational symmetry).

The squaring function has symmetry about the y-axis.

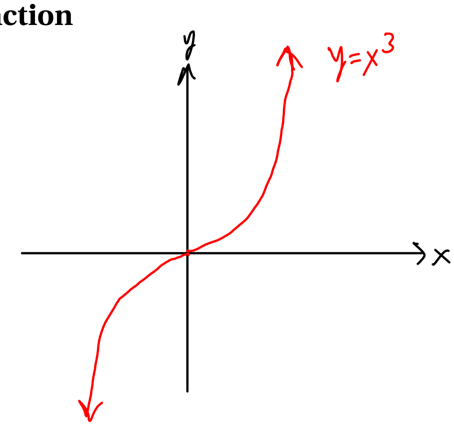
Symmetry and Even/Odd Functions

A function is <u>even</u> if and only if $f(-x) = f(x)$ for all x.
and <u>odd</u> if and only if $f(-x) = -f(x)$ for all x.

<u>Even</u> corresponds to symmetry about the <u>y-axis</u>.
<u>odd</u> corresponds to symmetry about the <u>origin</u>.

The Cubing Function

$f(x) = x^3$



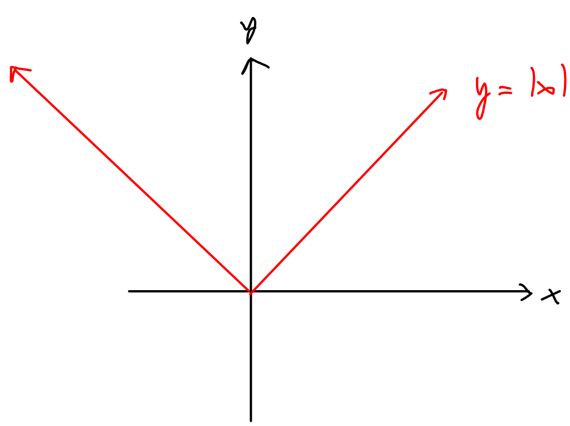
Symmetry of the Cubing Function

Analytically, $f(-x) = (-x)^3 = -x^3 = -f(x)$.
So $f(x) = x^3$ is <u>odd</u>.
Therefore the graph $y = x^3$ is symmetric about the origin.

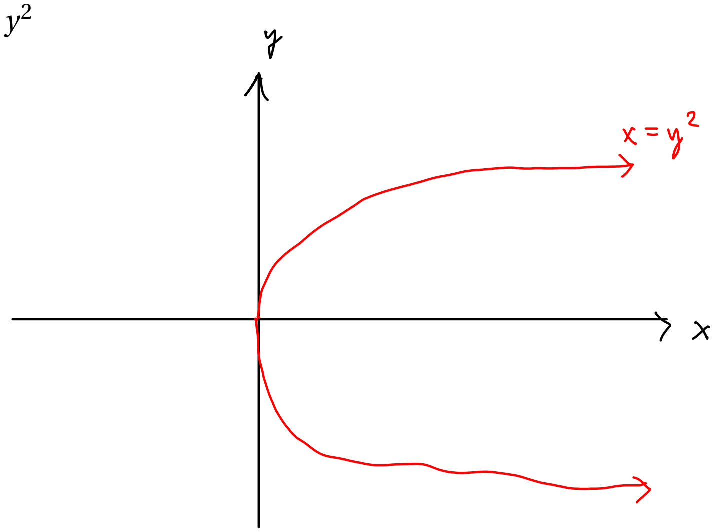
* Verify it!

The Absolute Value Function

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



The Relation $x = y^2$



Caution: NOT a function of x.

Symmetry about the x-axis

$x=y^2$ is symmetric about the x-axis because as a function of y, it is even.

As far as we are concerned, $x=y^2$ is not a function.

In this course, we want to consider functions of x.

The ONLY function with x-axis symmetry

. . . is very boring. $y=0$. Verify it.

Example Determine whether the functions are even, odd, or neither. What kind of symmetry to the graphs have (if any)?

(*a*) $f(x) = 13x^3 + x^5 - 6x$

$$f(-x) = 13(-x)^3 + (-x)^5 - 6(-x)$$
$$= -13x^3 - x^5 + 6x$$
$$= -(13x^3 + x^5 - 6x)$$
$$= -f(x)$$

$\boxed{\text{ODD}}$

(*b*) $g(x) = x^4 - 15x^2 + 12$

$$g(-x) = (-x)^4 - 15(-x)^2 + 12$$
$$= x^4 - 15x^2 + 12$$
$$= g(x)$$

$\boxed{\text{EVEN}}$

(*c*) $f(x) = \dfrac{x^3 + 12x}{2x^2 - 15}$

$$f(x) = \frac{(-x)^3 + 12(-x)}{2(-x)^2 - 15} = \frac{-x^3 - 12x}{2x^2 - 15} = -\frac{(x^3 + 12x)}{2x^2 - 15} = -f(x)$$

$\boxed{\text{odd}}$

(*d*) $h(x) = 2x^2 - 8x + 15$

$$h(x) = 2(-x)^2 - 8(-x) + 15$$
$$= 2x^2 + 8x + 15$$

$\boxed{\text{Neither}}$