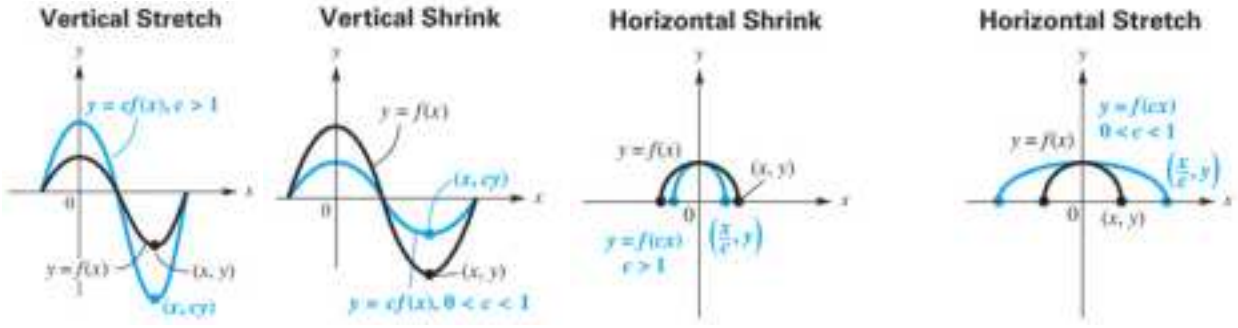
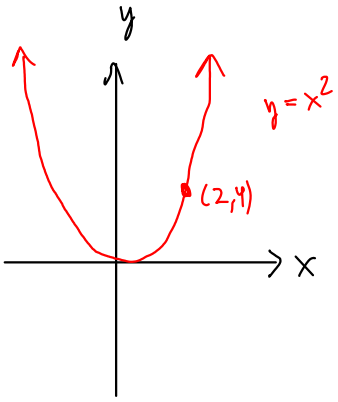
Section 2.3—Stretching, Shrinking, and Reflecting Graphs

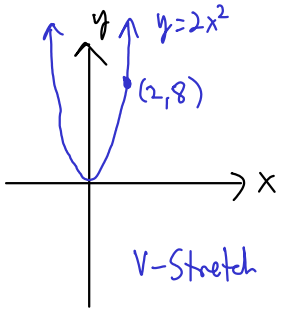
Let $c > 1$ be a positive constant that is larger than 1, and let $y = p(x)$ be a function with a known graph. The following table describes how to obtain the graph of a function $y = f(x)$ by using the graph of the parent function.

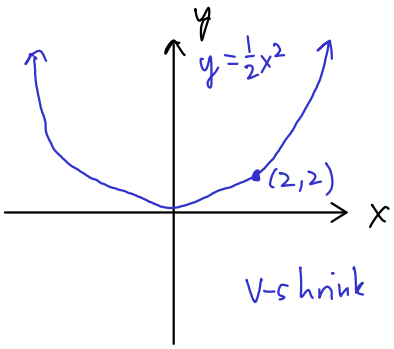
function	*effect on the parent graph*
$f(x) = c \cdot p(x)$	**vertical stretch** by a factor of c
$f(x) = \frac{1}{c} \cdot p(x)$	**vertical shrink** by a factor of c
$f(x) = p(c \cdot x)$	**horizontal shrink** by a factor of c
$f(x) = p\left(\frac{x}{c}\right)$	**horizontal stretch** by a factor of c

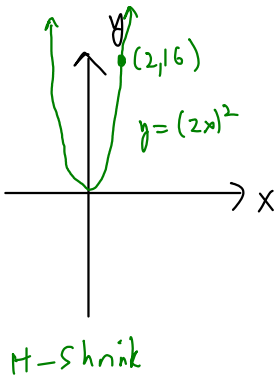


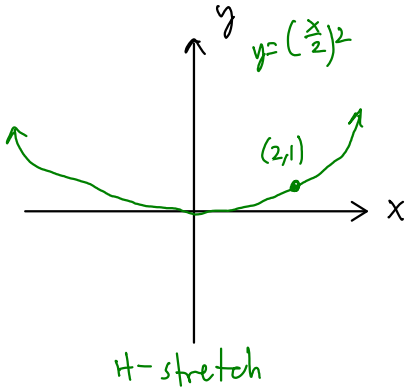
Example Let $c = 2$. Sketch the graphs of each possible transformation in the table above for the function $p(x) = x^2$.











Consider the function $f(x) = 2x^2 - 4x + 4$. Describe all transformations in the graph of the parent function.

$y = 2x^2 - 4x + 4$

Before we can complete the square

$y = 2(x^2 - 2x + 2)$

$y = 2((x^2 - 2x + 1) + 2(-1))$

$b = -2$

$\frac{b}{2} = -1$

$\left(\frac{b}{2}\right)^2 = 1$

$y = 2[(x-1)^2 + 1]$

So $\boxed{y = 2(x-1)^2 + 2}$

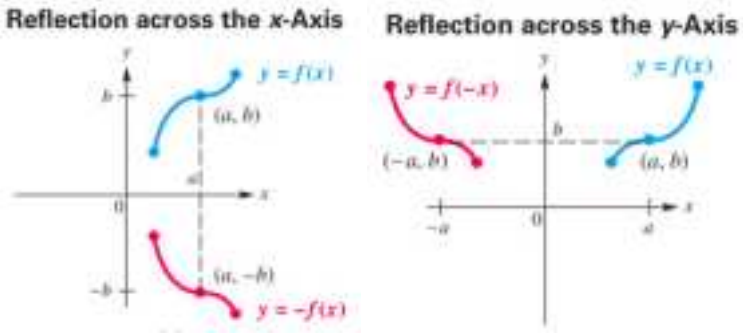
Parent function: $p(x) = x^2$

shifts: Right 1 unit
Up 2 units

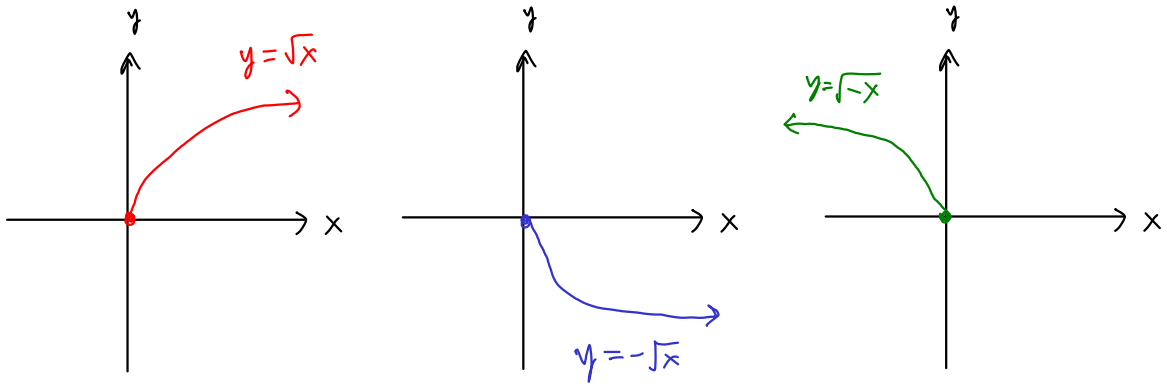
stretch/shrink: V-stretch by factor of 2

Let $y = p(x)$ be a function with a known graph. The following table describes how to obtain the graph of a function $y = f(x)$ by using the graph of the parent function.

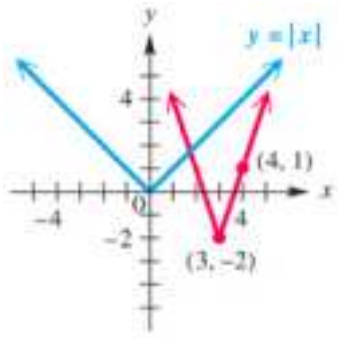
function	*effect on the parent graph*
$f(x) = -p(x)$	**reflection** over the x-axis
$f(x) = p(-x)$	**reflection** over the y-axis



Example Sketch the graphs of $y = -\sqrt{x}$ and $y = \sqrt{-x}$.



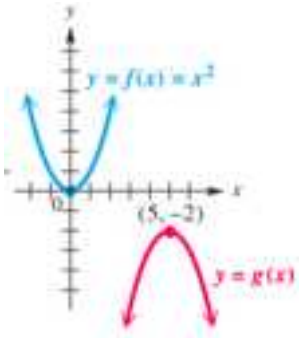
Example Describe the transformation and write an equation of the function defined by the graph.



(a)

2. Right 3
3. Down 2
1. V-stretch: 3

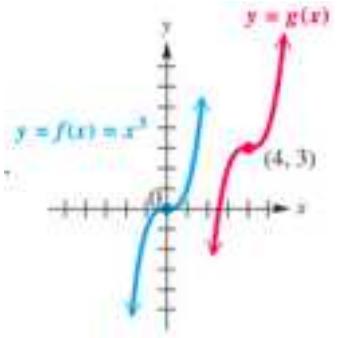
$\Rightarrow$ $f(x) = 3|x-3| - 2$



(b)

2. Right 5
3. down 2
1. V-reflection

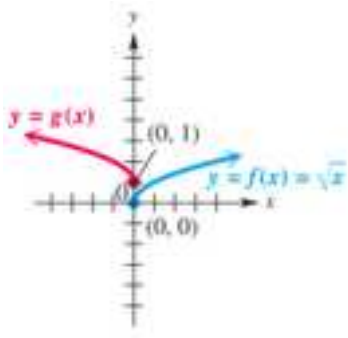
$\Rightarrow$ $f(x) = -(x-5)^2 - 2$



(c)

Up 3
Right 4

$f(x) = (x-4)^3 + 3$



(d)

H-flip
up 1

$f(x) = \sqrt{-x} + 1$

Example Write the equation in vertex form and describe the transformations of the graph.

$$y = -3x^2 - 9x + 3$$

Completing the square, again.

$y = -3\left(\left(x^2 + 3x + \frac{9}{4}\right) + \left(-1 - \frac{9}{4}\right)\right)$

$= -3\left[\left(x + \frac{3}{2}\right)^2 - \frac{13}{4}\right]$

So, $y = -3\left(x + \frac{3}{2}\right)^2 + \frac{39}{4}$

$b = 3$

$\frac{b}{2} = \frac{3}{2}$

$\left(\frac{b}{2}\right)^2 = \frac{9}{4}$

Transformations:

V-flip
V-stretch by factor of 3
Left 3/2 units
Up 39/4 units

In this order!!