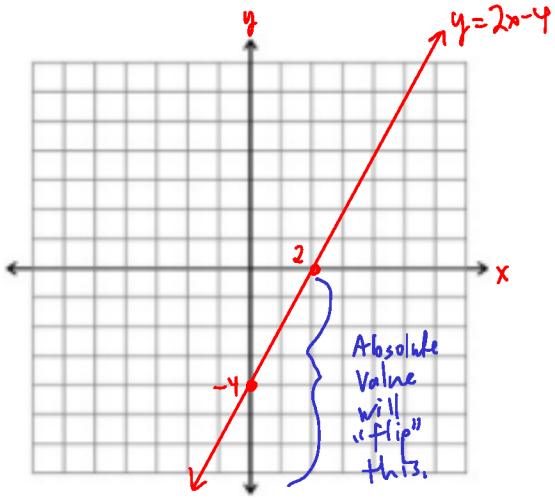
How to use this handout—This handout contains a skeleton of the notes that we will study in class this week. I've typed out definitions and theorems so that you don't have to exasperatedly copy what I'm writing, and populated these pages with a number of examples. My expectation of you is that you will fill in all of the details, ideas, *etc*, that I've left out.

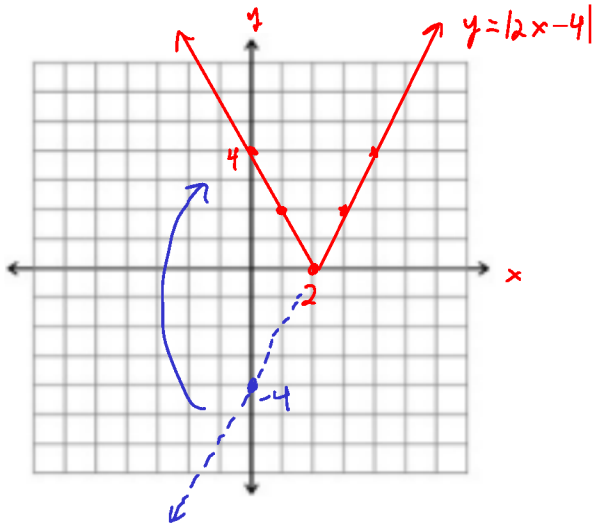
Section 2.4—Absolute Value Functions

Let f be a function with a known graph and consider the new function

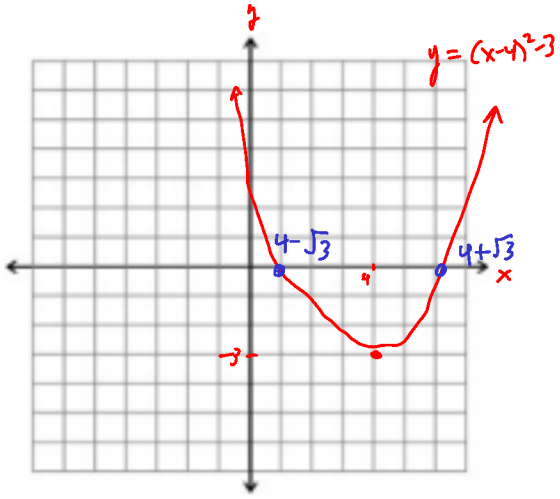
$$y = |f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}.$$

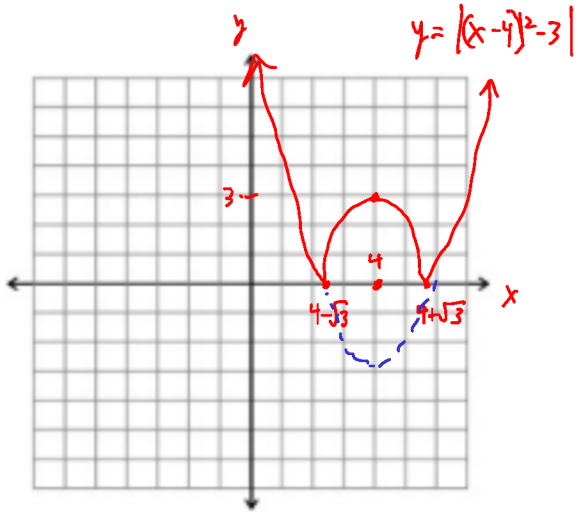
Example $y = |2x - 4|$





Example $g(x) = |(x-4)^2 - 3|$





$(x-4)^2 - 3 = 0$

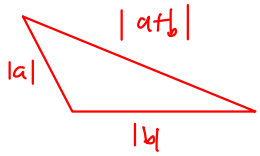
$(x-4)^2 = 3$

$x = 4 \pm \sqrt{3}$

Properties of Absolute Value

For real numbers a and b,

1. $|ab| =$ $|a| \cdot |b|$
2. $\left|\frac{a}{b}\right| =$ $\frac{|a|}{|b|}$ for $b \neq 0$.
3. $|-a| =$ $|a|$
4. Triangle Inequality: $|a| + |b| \geq |a+b|$



Example Solve $|2x+1| = 7$.

$2x+1=7$ AND/"OR" $2x+1=-7$

$2x=6$ $2x=-8$

$x=3$ $x=-4$

$x=-4, 3$

Solving Absolute Value Equations and Inequalities

Let k be a positive number. We interpret equations or inequalities involving absolute value symbols as

1. $|ax+b| = k$

   $ax+b=k$ or $ax+b=-k$

2. $|ax+b| > k$

   $ax+b>k$ or $ax+b<-k$

3. $|ax+b| < k$

   $-k < ax+b < k$

Example Solve and plot the solution.

(*a*) $|2x+1|>7$

$2x+1>7$ or $2x+1<-7$

$2x>6$ $2x<-8$

$x>3$ $x<-4$

(*b*) $|2x+1|<7$

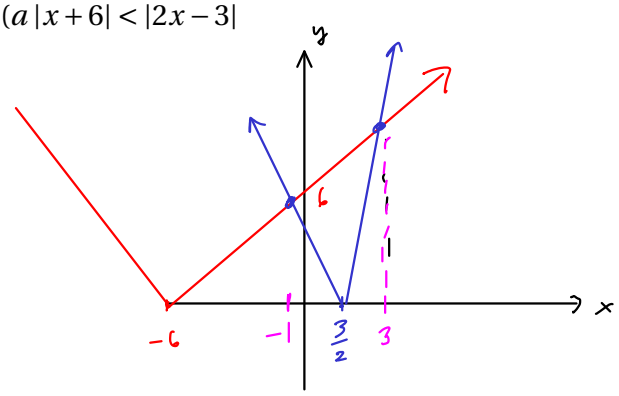
$-7<2x+1<7$

$-8<2x<6$

$-4<x<3$

Example Solve the inequalities and plot the solutions on a number line.

(*a* $|x+6|<|2x-3|$



$|2x-3| = |2(x-\frac{3}{2})| = 2|x-3/2|$

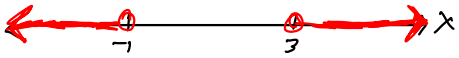
$x+6<-(2x-3)$ $x+6<2x-3$

$x+6<-2x+3$ $9<3x$

$3x<-3$ $3<x$

$x<-1$

or



(*b*) $|x+6|\geq|2x-3|$

Based on the graphs and an inspection:

$-1\leq x\leq 3$