MA 135: **College Algebra** (Fall 2016)
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Week 5 Notes

How to use this handout—This handout contains a skeleton of the notes that we will study in class this week. I've typed out definitions and theorems so that you don't have to exasperatedly copy what I'm writing, and populated these pages with a number of examples. My expectation of you is that you will fill in all of the details, ideas, *etc*, that I've left out.

Section 2.6—Operations on functions

Definition Let f and g be functions, and let x be in the domain of both f and g. Then the functions $f+g$, $f-g$, fg, $\frac{f}{g}$, and $f \circ g$ are defined as follows.

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(fg)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$

Example Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find $f+g$, $f-g$, fg, $\frac{f}{g}$, and $f \circ g$.

$(f+g)(x) = f(x) + g(x) = (x^2+1) + (3x+5) = x^2 + 3x + 6$

$(f-g)(x) = f(x) - g(x) = (x^2+1) - (3x+5) = x^2 - 3x - 4$

$(fg)(x) = (x^2+1)(3x+5) = 3x^3 + 5x^2 + 3x + 5$

$\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2+1}{3x+5}$, $3x+5 \neq 0 \Rightarrow x \neq \frac{-5}{3}$.

Example Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x-1}$. Find $f+g$, $f-g$, fg, $\frac{f}{g}$, and $f \circ g$. What are their domains?

$f(x) + g(x) = 8x - 9 + \sqrt{2x-1}$

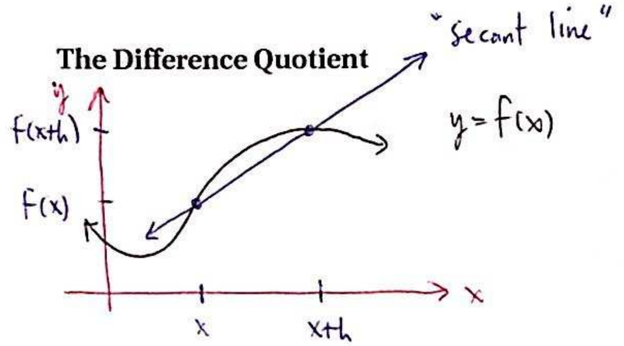
$f(x) - g(x) = 8x - 9 - \sqrt{2x-1}$

$f(x) \cdot g(x) = (8x-9)\sqrt{2x-1} = 8x\sqrt{2x-1} - 9\sqrt{2x-1}$

$\Bigg\}$ $x \geq \frac{1}{2}$.

$\dfrac{f(x)}{g(x)} = \dfrac{8x-9}{\sqrt{2x-1}}$, $x > \frac{1}{2}$.

The Difference Quotient



The <u>difference quotient</u> of f is the slope of the secant line,

$$\boxed{DQ = \frac{f(x+h) - f(x)}{h}}$$

Example Find the difference quotient for $f(x) = x^2$.

$$f(x+h) = (x+h)^2 = \cancel{x^2} + 2xh + h^2$$
$$- f(x) = \cancel{x^2}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} \qquad \boxed{DQ = 2x + h}$$

Example Find the difference quotient for $y = x^4$.

$$f(x+h) = (x+h)^4 = \cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$
$$- f(x) = \cancel{x^4}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\cancel{h}(4x^3 + 6x^2h + 4xh^2 + h^3)}{\cancel{h}} \qquad \boxed{DQ = 4x^3 + 6x^2h + 4xh^2 + h^3}$$

Example Find the difference quotient for $f(x) = \dfrac{1}{x}$.

$$f(x+h) = \frac{1}{x+h}$$
$$- f(x) = \frac{1}{x}$$

$$f(x+h) - f(x) = \frac{1 \cdot x}{x(x+h)} - \frac{1(x+h)}{x(x+h)} = \frac{x - (x+h)}{x(x+h)} = \frac{-h}{x(x+h)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{h}\left(f(x+h) - f(x)\right)$$
$$= \frac{1}{\cancel{h}}\left(\frac{-\cancel{h}}{x(x+h)}\right)$$

so, $\boxed{DQ = \dfrac{-1}{x(x+h)}}$

Example Find the difference quotient for $f(x) = \sqrt{x}$.

$$f(x+h) = \sqrt{x+h}$$
$$- f(x) = \sqrt{x}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}} = DQ}$$