Section 3.2—Quadratic Functions and Graphs

Standard Form

$f(x) = ax^2 + bx + c, \qquad a \neq 0$

Vertex Form

$f(x) = a(x-h)^2 + k \qquad a \neq 0$ is the same as standard form.

(h,k) is the vertex.

Example Write the quadratic function in vertex form.

$$P(x) = 2x^2 + 4x - 16$$

$$P(x) = 2(x^2 + 2x + 1 - 8 - 1)$$

$$= 2((x+1)^2 - 9)$$

$$\boxed{P(x) = 2(x+1)^2 - 18}$$

Example Sketch the graph of the function $P(x) = -(x+3)^2 + 1$. Label the vertex, x- and y-intercepts, and the axis so symmetry.

$P(x) = -(x+3)^2 + 1$

vertex: $(-3, 1)$

y-int: $y = -(0+3)^2 + 1$

$= -9 + 1$

$= -8$

$(0, -8)$

x-int: $0 = -(x+3)^2 + 1$

$(x+3)^2 = 1$

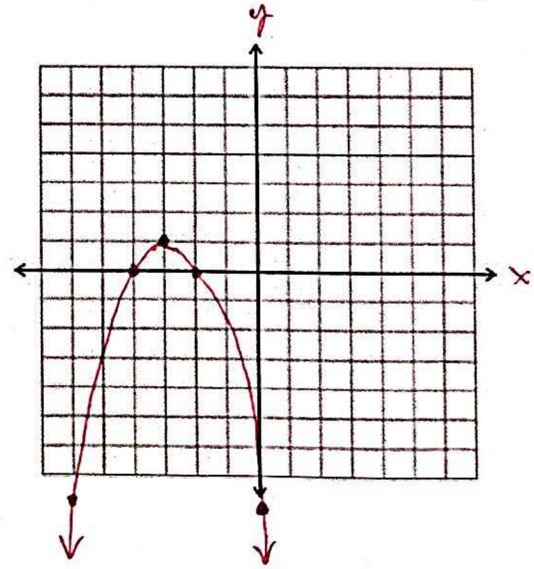
$x + 3 = \pm 1$

$x = -3 \pm 1$

$x = -4, -2$

$(-4, 0)$

$(-2, 0)$



$y = -(x+3)^2 + 1$

The Vertex Formula

Let $P(x) = ax^2 + bx + c$.

The <u>vertex</u> is given by

$$\boxed{V = \left(\frac{-b}{2a}, P\left(\frac{-b}{2a}\right)\right)}$$

Extreme Values

If $a > 0$, then the parabola opens upward and the vertex is the lowest point. So $P\left(\frac{-b}{2a}\right)$ is the <u>minimum</u> value of the function.

If $a < 0$, then $P\left(\frac{-b}{2a}\right)$ is the <u>maximum</u> value of the function.

Example Give the coordinates of the extreme point of the graph and determine whether it is a maximum or minimum.

$$P(x) = -x^2 - 6x - 8$$

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3$$

$$\begin{aligned} P(-3) &= -(-3)^2 - 6(-3) - 8 \\ &= -9 + 18 - 8 \\ &= 1 \end{aligned}$$

$\boxed{\text{The vertex } (-3,1) \text{ is the maximum point on the parabola.}}$

Curve Fitting

Example Find the equation of the parabola with vertex $(2, -3)$ and passing through the point $(8, 69)$.

$$\left.\begin{aligned} &P(x) = a(x-h)^2 + k \\ &(h,k) = (2,-3) \\ &Q(x,y) = (8,69) \end{aligned}\right\} \Rightarrow$$

$$\begin{aligned} P(x) &= a(x-2)^2 - 3 \\ 69 &= a(8-2)^2 - 3 \\ 72 &= a(6)^2 \\ 72 &= 36a \\ a &= 2 \end{aligned}$$

$$\Rightarrow \boxed{P(x) = 2(x-2)^2 - 3}$$