MA 135: College Algebra (Fall 2016)
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Week 8 Notes

How to use this handout—This handout contains a skeleton of the notes that we will study in class this week. I've typed out definitions and theorems so that you don't have to exasperatedly copy what I'm writing, and populated these pages with a number of examples. My expectation of you is that you will fill in all of the details, ideas, *etc*, that I've left out.

Section 3.5—Higher Degree Polynomials

General polynomial functions

A polynomial function of degree n is of the form

$$\boxed{P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0}$$

where a_i are real numbers, called the coefficients.
a_n is called the leading coefficient
$a_n x^n$ is the leading term.

Example Cubic functions

A cubic function is a polynomial of degree 3.

$$\boxed{P(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0}$$

Example Quartic functions

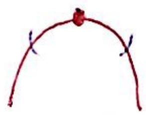
A quartic function is a polynomial of degree 4.

$$\boxed{P(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0}$$

Absolute and Relative Extrema

A polynomial has extreme (maximum or minimum) points at "turning points".

Locally:



local (relative) max

local (relative) min

These are called critical points of the polynomial.

Globally:

local extrema may not be global (absolute) extrema.

But a local extreme point that is also a global extreme point is called an absolute max or min.

Example $f(x) = x^4 - 2x^2$

Classify.
Graph on computer.
Identify local/global extrema.

Example $P(x) = x^3 - 2x^2 + x - 2$

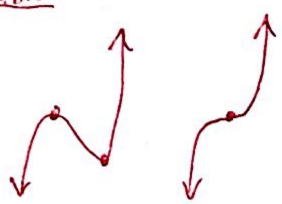
Repeat.

Number of Turning Points

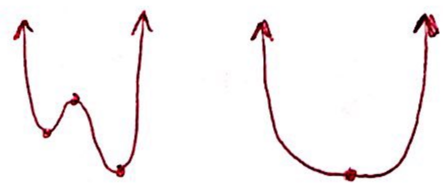
Let $y = P(x)$ be a polynomial of degree n. Then the graph of P can have at most $n-1$ turning points.

Ex.

Cubic:



Quartic:



Example $f(x) = x^4 - x^2 + 5x - 4$

Classify.

Graph on Computer.

Observe/ Analyze.

End Behavior

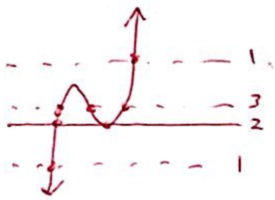
Let $y = P(x)$ be a polynomial of degree n.

n	a_n	end behavior
even	$a_n > 0$	↖ . . . ↗
even	$a_n < 0$	↙ . . . ↘
odd	$a_n > 0$	↙ . . . ↗
odd	$a_n < 0$	↖ . . . ↘

x-intercepts (Real Zeros)

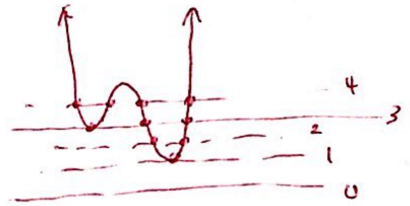
A polynomial of degree n can have at most n distinct real zeros.
These x-values represent the x-values of x-intercepts.

odd degree:



Odd-degree polynomials have at least one real zero.

Even degree:



Number of x-intercepts

Analyzing a Polynomial Function $P(x) = x^5 + 2x^4 - x^3 + x^2 - x - 4$

$P(x) = (x+1)(x^4 + x^3 - 2x^2 + 3x - 4)$ by synthetic division.

So $x = -1$ is a real zero.

There are two other non-integer zeros that can be seen by looking at the graph.