Section 3.8—Polynomial Equations and Inequalities

25. Example Solve $x^3+3x^2-4x-12=0$ by factoring.

$$x^2(x+3)-4(x+3)=0$$

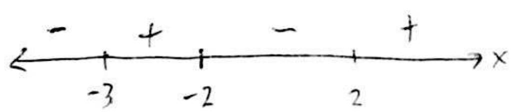
$$(x^2-4)(x+3)=0$$

$$(x+2)(x-2)(x+3)=0$$

so $\boxed{x=-3,-2,2}$

26. Example Now solve $x^3+3x^2-4x-12>0$.

we want to solve $(x+2)(x-2)(x+3)>0$



Choose test values.

so the solution is

$$(-3,-2)\cup(2,\infty)$$

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27. Example Solve $x^4-6x^2-40=0$.

let $u=x^2$, so $x^4-6x^2-40=u^2-6u-40=0$

$$(u-10)(u+4)=0$$

$u=10$ $\quad u=-4$

$x^2=10$ $\quad x^2=-4$

$\boxed{x=\pm\sqrt{10} \quad x=\pm 2i}$

28. Example Solve $2x^3 + 12 = 3x^2 + 8x$.

$$2x^3 - 3x^2 - 8x + 12 = 0$$

$$x^2(2x-3) - 4(2x-3) = 0$$

$$(x^2-4)(2x-3) = 0$$

$$(x+2)(x-2)(2x-3) = 0$$

$$\boxed{x = -2,\ 2,\ \tfrac{3}{2}}$$

29. Example Show that 2 is a root of the function

$$P(x) = x^3 + 3x^2 - 11x + 2,$$

and use this information to solve $x^3 + 3x^2 - 11x + 2 \le 0$.

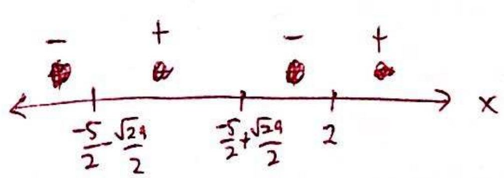
$$\begin{array}{r|rrrr} 2 & 1 & 3 & -11 & 2 \\ & \downarrow & 2 & 10 & -2 \\ \hline & 1 & 5 & -1 & \boxed{0} \end{array}$$

$$P = (x-2)(x^2+5x-1) = (x-2)\left(x + \tfrac{5}{2} + \tfrac{\sqrt{29}}{2}\right)\left(x + \tfrac{5}{2} - \tfrac{\sqrt{29}}{2}\right)$$

Complete the square: $\left(x^2 + 5x + \frac{25}{4}\right) - 1 - \frac{25}{4} = 0$

$$\left(x + \tfrac{5}{2}\right)^2 - \tfrac{29}{4} = 0$$

$$x = \tfrac{-5}{2} \pm \tfrac{\sqrt{29}}{2}$$



So, the sol'n is:

$$\boxed{\left(-\infty, \tfrac{-5}{2} - \tfrac{\sqrt{29}}{2}\right] \cup \left[\tfrac{-5}{2} + \tfrac{\sqrt{29}}{2}, 2\right]}$$

30. Example Find all 6 complex sixth roots of 64; *i.e.*, compute $\sqrt[6]{64}$.

$$x = \sqrt[6]{64}$$

$$x^6 = 64$$

$$x^6 - 64 = 0$$

$$(x^3 - 8)(x^3 + 8) = 0$$

$$(x-2)(x^2+2x+2)(x+2)(x^2-2x+2) = 0$$

$$x^2 + 2x + 2 = 0 \qquad\qquad x^2 - 2x + 2 = 0$$

$$x^2 + 2x + 1 = -2 + 1 \qquad\qquad x - 2x + 1 = -2 + 1$$

$$(x+1)^2 = -1 \qquad\qquad (x-1)^2 = -1$$

$$x = -1 \pm i \qquad\qquad x = 1 \pm i$$

So the 6 complex 6th roots of 64 are

$$\boxed{x = 2, -2, -1+i, -1-i, 1+i, 1-i}$$

Stopping Point This is the end of the material for Exam 3.