MA 135: College Algebra (Fall 2016)
Instructor: Justin Ryan
Week 10 Notes

How to use this handout—This handout contains a skeleton of the notes that we will study in class this week. I've typed out definitions and theorems so that you don't have to exasperatedly copy what I'm writing, and populated these pages with a number of examples. My expectation of you is that you will fill in all of the details, ideas, *etc*, that I've left out.

Section 4.2—Rational Functions

1. Rational Functions

A rational function is of the form $R(x) = \frac{P(x)}{Q(x)}$ where P and Q are polynomials.

So, $R(x) = \dfrac{a_n x^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \dots + b_2 x^2 + b_1 x + b_0}$

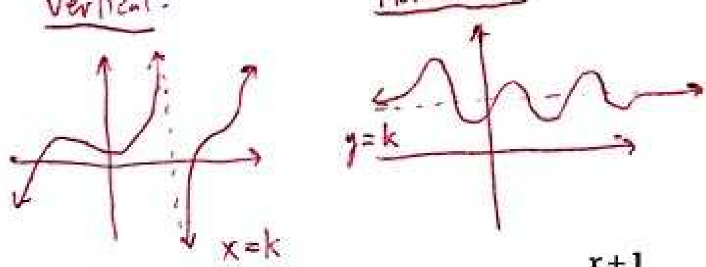
The domain of R is all x-values except those that are roots of the denominator Q; $Q(x) \neq 0$.

R is continuous on its entire domain: $Q(x) \neq 0$.

2. Vertical and Horizontal Asymptotes

An asymptote is a line that the graph approaches w/out equalling.

Vertical: Horizontal:



To Find them:

Vertical: happen at roots of Q that are not roots of P: $Q(x)=0$, $P(x) \neq 0$.

Horizontal: Arrange that the leading terms of P and Q are of the same degree by adding a "$0x^r$" term where necessary, $r = m$ or n. Then put $y =$ the ratio of leading coefficients.

3. Finding Asymptotes by Hand $f(x) = \dfrac{x+1}{2x^2+5x-3}$

H.A.: $f(x) = \dfrac{0x^2 + x + 1}{2x^2 + 5x - 3}$, so $\boxed{y=0}$ is an H.A.

V.A.: $2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$

$2x(x+3) - 1(x+3) = 0$

$(2x-1)(x+3) = 0$

$x = \frac{1}{2}$ $x = -3$

Neither $\frac{1}{2}$ nor -3 make the top $=0$, so the VA's are $\boxed{x = \frac{1}{2}}$ and $\boxed{x=-3}$.

4. Example $f(x) = \dfrac{2x+1}{x-3}$

H.A. $y = \frac{2}{1} = 2.$

V.A: $x = 3$

5. Another Example $f(x) = \dfrac{x^2+1}{x-2}$

H.A. $f(x) = \dfrac{1x^2 + 1}{0x^2 + x - 2}$

$y = \frac{1}{0}$ is undefined, so no HA.

V.A. $x - 2 = 0 \Rightarrow x = 2.$

6. Oblique (or Slant) Asymptotes $f(x) = \dfrac{x^2+1}{x-2}$

A slant asymptote is a line that is neither vertical nor horizontal, but that the graph approaches as $x \to \pm\infty$.

~~[illegible]~~ These only occur when the degree of the numerator is exactly one larger than the degree of the dominator. To find it, divide and take the quotient.

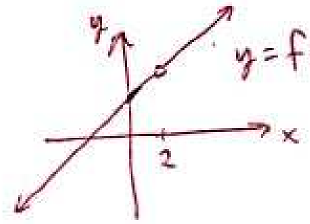
$$\frac{x^2+1}{x-2} = x + 2 + \frac{5}{x-2}$$

$$\begin{array}{r|rrr} 2 & 1 & 0 & 1 \\ & & 2 & 4 \\ \hline & 1 & 2 & \boxed{5} \end{array}$$

The slant asymptote is $\boxed{y = x+2}$. Indeed, as $x \to \pm\infty$, $\frac{5}{x-2} \to 0$.

7. Holes … as opposed to asymptotes. $f(x) = \dfrac{x^2-4}{x-2} = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = x+2, \quad x \neq 2.$

A <u>hole</u> occurs where P and Q have a common factor (of the same multiplicity).
Indeed, the graph of this function looks like the line $y = x+2$, but w/ a single point removed.



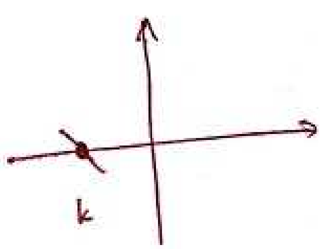
8. Behavior Near Asymptotes

We need to determine how the graph of a rational function behaves near an asymptote. This can be done by choosing test values: very close to the asymptote for VAs, and very large (positive and negative) for HAs and SAs.

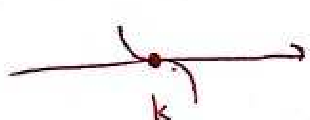
9. Behavior Near x-Intercepts

The behavior near x-intercepts is more subtle. Suppose $(x-k)^r$ is a factor of P that is not a factor of Q. Here $r \geq 1$ is an integer and k is the root of R (hence also P). There are 3 cases.

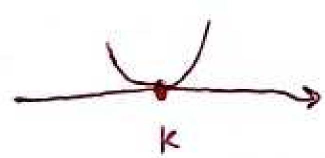
$r=1$: $(x-k)$



r odd, $r>1$: $(x-k)^3$



r even



10. Example Find the domain of f, all x- and y-intercepts, all vertical, horizontal, and slant asymptotes. Plot selected points as necessary, and sketch the graph of the function. **After** you've sketched it by hand, check your work using a graphing utility.

$$f(x) = \frac{x+1}{2x^2+5x-3}$$

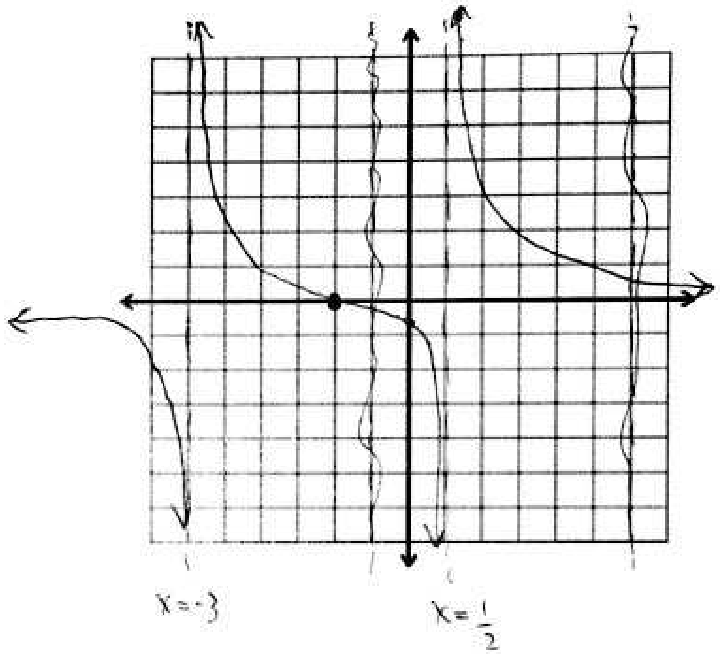
domain: $x \neq \frac{1}{2}, -3$

x-int: $(-1, 0)$

y-int: $(0, -\frac{1}{3})$

H.A.: $y = 0$

V.A.: $x = \frac{1}{2}$, $x = -3$



11. Example Find the domain of f, all x- and y-intercepts, all vertical, horizontal, and slant asymptotes. Plot selected points as necessary, and sketch the graph of the function. **After** you've sketched it by hand, check your work using a graphing utility.

$$f(x) = \frac{2x+1}{x-3}$$

domain: $x \neq 3$

x-int: $x = -\frac{1}{2} \Rightarrow (-\frac{1}{2}, 0)$

y-int: $(0, -\frac{1}{3})$

H.A.: $y = 2$

V.A.: $x = 3$