MA 135: **College Algebra** (Fall 2016)
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Week 11 Notes

How to use this handout—This handout contains a skeleton of the notes that we will study in class this week. I've typed out definitions and theorems so that you don't have to exasperatedly copy what I'm writing, and populated these pages with a number of examples. My expectation of you is that you will fill in all of the details, ideas, *etc*, that I've left out.

Section 4.3—Rational Equations

1. Solving a Rational Equation

Follow these steps:

1. Determine all points where the rational expression is undefined.
2. Clear the denominators to obtain a polynomial expression.
3. Solve the polynomial.
4. Reject any "solutions" that coincide w/ values from step 1.

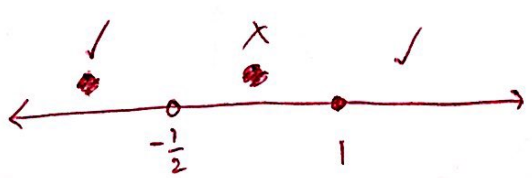
2. Example $\dfrac{x+2}{2x+1} = 1$

$2x+1 \neq 0$
$x \neq -\frac{1}{2}$

$x+2 = 2x+1$
$\boxed{x = 1.}$

3. Example $\dfrac{x+2}{2x+1} \leq 1$

Check intervals defined by both sets of points found in Ex 2.



test values:

$x=-1$: $\frac{1}{-1} = -1 < 1$

$x = 0$: $\frac{2}{1} = 2 > 1$

$x = 2$: $\frac{4}{5} < 1$

So the solution is

$\boxed{(-\infty, -\tfrac{1}{2}) \cup [1, \infty)}.$

4. Example $\frac{x}{x-2} + \frac{1}{x+2} = \frac{8}{x^2-4}$

$\boxed{x \neq \pm 2}$

$$\frac{x(x+2) + 1(x-2)}{\cancel{x^2-4}} = \frac{8}{\cancel{x^2-4}}$$

$$x^2 + 2x + x - 2 = 8$$

$$x^2 + 3x - 10 = 0$$

$$(x+5)(x-2) = 0$$

$$\boxed{x = -5} \quad x = \cancel{2}$$

only solution.

5. Example Consider the rational function $f(x) = 7x^{-4} - 8x^{-2} + 1$. (*a.*) Find the roots of f. (*b.*) Use a graphing utility to graph $y = f(x)$, and identify the roots of f. (*c.*) Use the graph to solve the inequalities $f(x) \leq 0$ and $f(x) > 0$.

a) $f(x) = \frac{7}{x^4} - \frac{8}{x^2} + 1 = \frac{7 - 8x^2 + x^4}{x^4}$

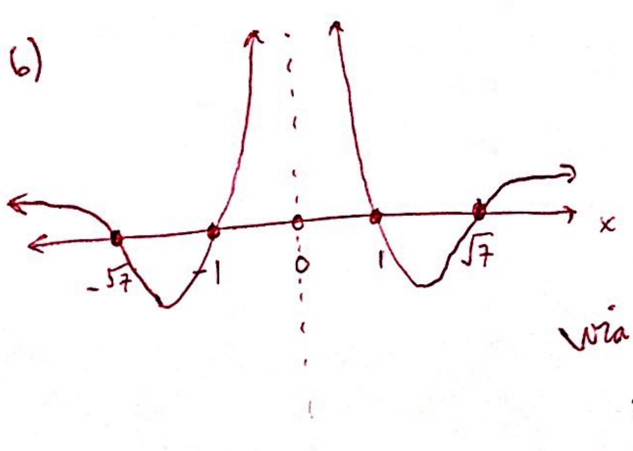
$\underline{\text{domain:}}$ $x \neq 0$

$$x^4 - 8x^2 + 7 = 0$$

$$(x^2 - 7)(x^2 - 1) = 0$$

a) $\boxed{x = \pm\sqrt{7} \quad x = \pm 1}$

b)



via geogebra.org.

c) $f(x) \leq 0: [-\sqrt{7}, -1] \cup [1, \sqrt{7}]$

$f(x) > 0: (-\infty, -\sqrt{7}) \cup (-1, 0) \cup (0, 1) \cup (\sqrt{7}, \infty)$