Math 242 – Calc I 8/21

§1.5 – Limits

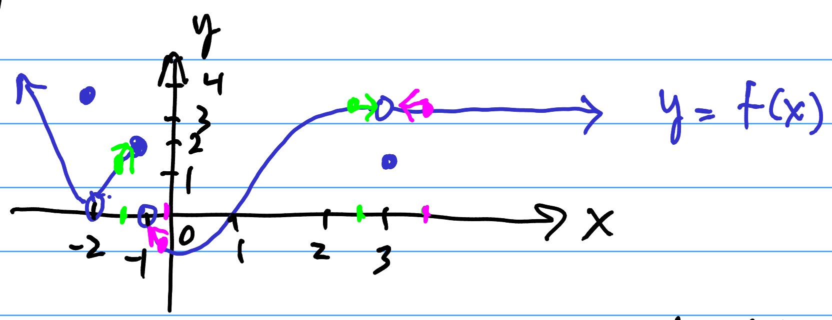
Defn. Let f be a function that is defined on some open interval containing the point a, but not necessary at a. If we can make the values of f arbitrarily close to a number L by making x become arbitrarily close to a, then we say

"the limit of f as x approaches a is L".

We'll write,

$$\lim_{x\to a} f(x) = L$$

Ex. Graphical



dom(f) = ℝ

$f(3) = 1$ $\qquad \lim_{x\to 3} f(x) = 3$

$f(1) = 0$ $\qquad \lim_{x\to 1} f(x) = 0$

$f(-1) = 2$

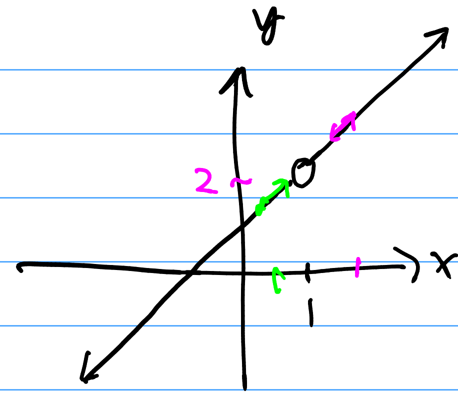
$\left.\begin{array}{l}\lim_{x\to -1^+} f(x) = 0 \\ \lim_{x\to -1^-} f(x) = 2\end{array}\right\}$ Does **NOT** exist, DNE

Ex. $f(x) = \dfrac{x^2-1}{x-1}$ $\qquad f(1) = \frac{0}{0} = \text{undef.}$

$$\lim_{x\to 1} f(x) = ? = \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

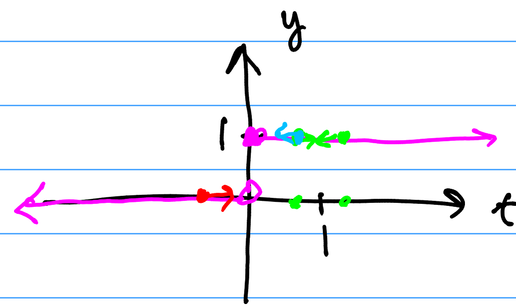
$$= \lim_{x\to 1} (x+1) = 1+1 = 2$$

$$\boxed{f(x) = \frac{x^2-1}{x-1}} = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = x+1, \quad x \neq 1$$



Ex. Heaviside Function

$$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



$H(1) = 1$

$\lim_{t \to 1} H(t) = 1$

$H(0) = 1$

$\lim_{t \to 0^-} H(t) = 0$

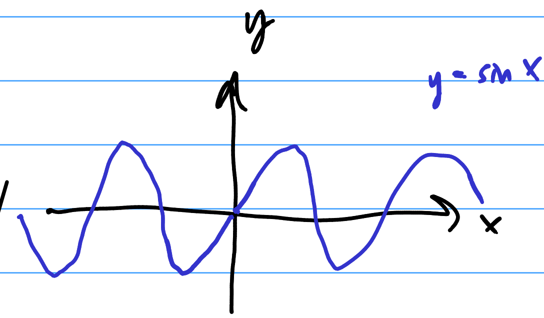
$\lim_{t \to 0^+} H(t) = 1$

$$\boxed{\lim_{t \to 0} H(t) = DNE}$$

Ex. Topologist's Sine Curve

$$f(x) = \sin\left(\frac{\pi}{x}\right)$$

Recall:



$y = \frac{\pi}{x}$ $\qquad \frac{\pi}{0} = \text{undef.}$

$$\lim_{x \to 0^+} \frac{\pi}{x} = \frac{\pi}{0^+} = +\infty \text{ or DNE}$$

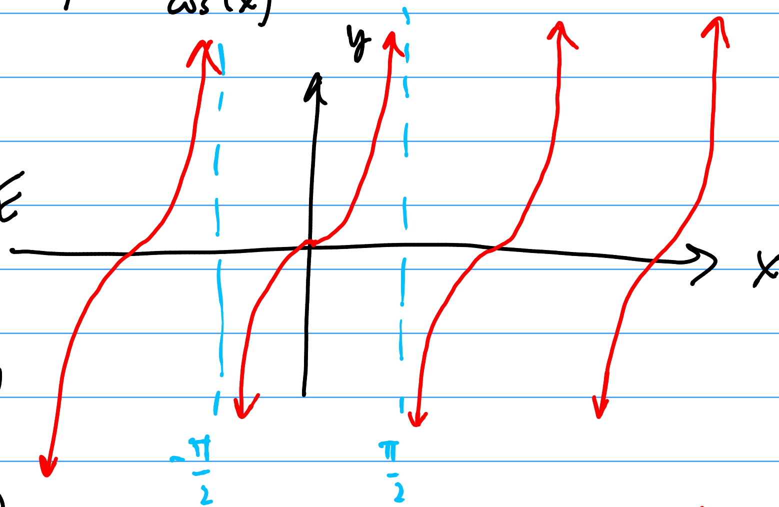
$$\lim_{x \to 0^+} \sin\left(\frac{\pi}{x}\right) = \lim_{u \to \infty} \sin(u) = DNE$$

Ex. $y = \tan(x) = \frac{\sin(x)}{\cos(x)}$

$\lim_{x \to \frac{\pi}{2}} \tan(x) = DNE$

$\lim_{x \to \frac{\pi}{2}^-} \tan(x) = +\infty$

$\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$

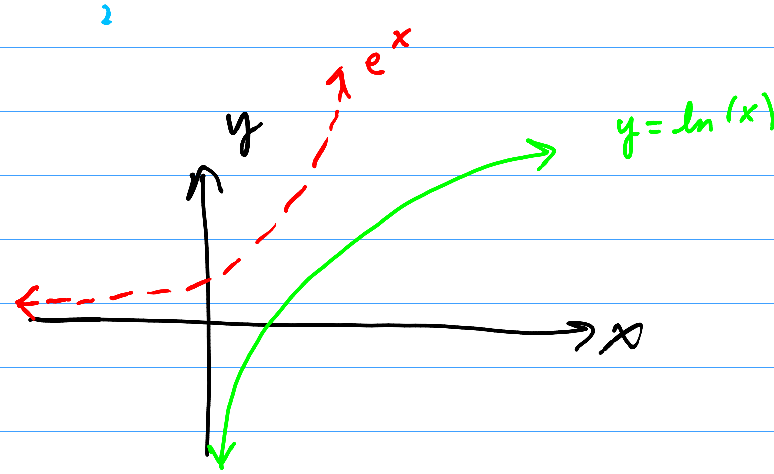


Ex. $y = \ln(x)$

$y = \ln(x) \Leftrightarrow e^y = x$

$dom(\ln(x)) = (0, \infty)$

$\lim_{x \to 0^+} \ln(x) = -\infty$



Ex. $f(x) = \frac{2x}{x-3}$

set builder ↙ interval ↓

$dom(f)$: $\{x \mid x \neq 3\}$ or $(-\infty, 3) \cup (3, \infty)$

$\lim_{x \to 3} f(x) = ?$