M242 - Calc I 8/22

§1.5

Ex. $\lim_{x\to 2} \frac{x^2-5x+6}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{x-2}} = \lim_{x\to 2} x-3$

$= 2-3 = -1$

Ex. $\lim_{x\to 0} \frac{\sin x}{x} = 1$ by a table.

Ex. $\lim_{x\to 2} \frac{x^2+4}{x-2} = \frac{4+4}{0} = \frac{8}{0} = \pm\infty$ = DNE

Ex. $\lim_{h\to 0} \frac{(2+h)^2-4}{h} = \frac{0}{0}$ Do more work!

$\lim_{h\to 0} \frac{(h^2+4h+\cancel{4})-\cancel{4}}{h} = \lim_{h\to 0} \frac{h^2+4h}{h} = \lim_{h\to 0} \frac{\cancel{h}(h+4)}{\cancel{h}}$

$= \lim_{h\to 0} h+4 = 0+4 = 4$

Ex. In relativity,

$$m(v) = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

m_0 = constant

c = speed of light

$\lim_{v\to c^-} m(v) = ? = \boxed{\lim_{v\to c^-} \frac{m_0}{\sqrt{1-v^2/c^2}}} = \frac{m_0}{\sqrt{1-(c^-/c)^2}}$

since $c^- < c$, then $\underbrace{c^-/c}_{1^-} < 1$

$= \frac{m_0}{\sqrt{1-1^-}}$

$= \frac{m_0}{\sqrt{0^+}} = \frac{m_0}{0^+}$

$= \begin{cases} +\infty & \text{if } m_0 > 0 \\ 0 & \text{if } m_0 = 0 \end{cases}$

§1.6 – Calculating Limits w/ Limit Laws

Thm. Let c be a constant and suppose the limits exist

$$\lim_{x\to a} f(x) = L \quad \text{and} \quad \lim_{x\to a} g(x) = M$$

1. $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = L \pm M$
2. $\lim_{x\to a} [c f(x)] = c \lim_{x\to a} f(x) = c \cdot L$
3. $\lim_{x\to a} [f(x)\, g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right) = LM$
4. $\lim_{x\to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$ as long as $M \neq 0$!
5. $\lim_{x\to a} (f(x))^n$ for $n \geq 0$ is an integer.

$$= \left(\lim_{x\to a} f(x)\right)^n = L^n.$$

6. $\lim_{x\to a} c = c$
7. $\lim_{x\to a} x = a \longrightarrow \lim_{x\to a} x^n = a^n$ $\quad n > 0$ positive integer
8. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ as long as $a \geq 0$ when n is even.

Ex. $\lim_{x\to 5} (2x^2 - 3x + 4) = \lim_{x\to 5} 2x^2 - \lim_{x\to 5} 3x + \lim_{x\to 5} 4$

$$= 2 \lim_{x\to 5} x^2 - 3 \lim_{x\to 5} x + \lim_{x\to 5} 4$$

$$= 2(5^2) - 3(5) + 4 = 39$$