M242 8/23

§1.6, cont'd

Thm. Direct Substitution: If f is a polynomial or rational function and a is any point in the domain of f, then

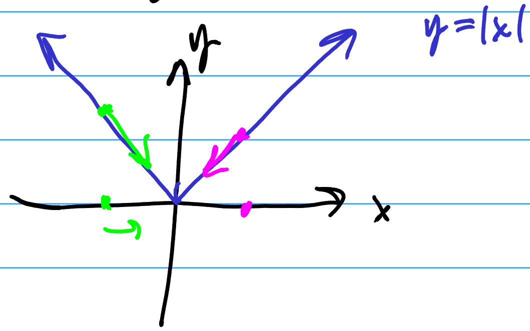
$$\lim_{x \to a} f(x) = f(a)$$

Ex. $\lim_{x \to 1} g(x)$ where $g(x) = \begin{cases} x+1 & x \neq 1 \\ \pi & x = 1 \end{cases}$

$g(1) = \pi$

$\lim_{x \to 1} g(x) = \lim_{x \to 1} x+1 = 1+1 = 2$

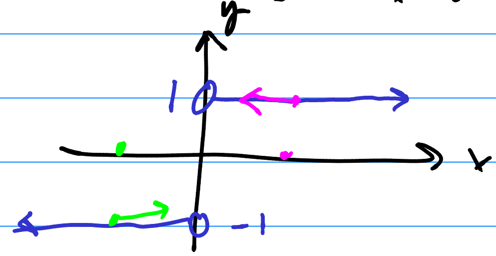
Ex. $\lim_{x \to 0} |x| = 0$



$\lim_{x \to 0} \frac{|x|}{x}$

$\frac{|x|}{x} = \begin{cases} x/x & x \geq 0 \\ -x/x & x < 0 \end{cases}$

$\frac{|x|}{x} = \begin{cases} 1 & x > 0 \\ \text{undef} & x = 0 \\ -1 & x < 0 \end{cases}$



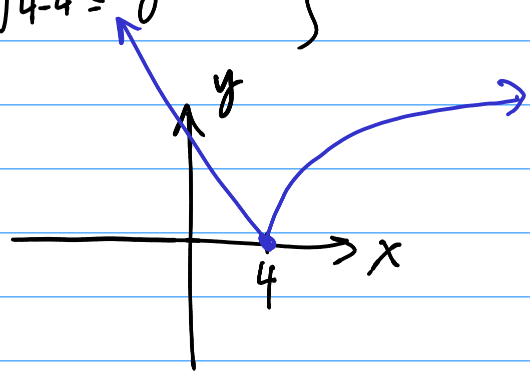
$\left.\begin{array}{l} \lim_{x \to 0^-} \frac{|x|}{x} = -1 \\ \lim_{x \to 0^+} \frac{|x|}{x} = 1 \end{array}\right\} \neq$

$\lim_{x \to 0} \frac{|x|}{x} = \text{DNE}$

Ex. $f(x) = \begin{cases} \sqrt{x-4} & x > 4 \\ 8-2x & x \le 4 \end{cases}$ $\qquad \lim_{x \to 4} f(x) = ?$

$$\left.\begin{aligned} \lim_{x \to 4^-} f(x) &= \lim_{x \to 4^-} 8-2x = 8-2(4) = 0 \\ \lim_{x \to 4^+} f(x) &= \lim_{x \to 4^+} \sqrt{x-4} = \sqrt{4-4} = 0 \end{aligned}\right\}$$

$$\lim_{x \to 4} f(x) = 0$$



Thm. Squeeze Theorem. If $f(x) \le g(x) \le h(x)$ when x is near a, except possibly when $x = a$, and

$$\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x),$$

then $\lim_{x \to a} g(x) = L$.

Ex. $g(x) = x^2 \sin\left(\frac{1}{x}\right)$ $\qquad \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

we know $\sin\left(\frac{1}{x}\right)$ is always between -1 and 1

$$x^2\left(-1 \le \sin\left(\tfrac{1}{x}\right) \le 1\right) \qquad \text{for all } x$$

$$\underset{f(x)}{-x^2} \le \underbrace{x^2 \sin\left(\tfrac{1}{x}\right)}_{g(x)} \le \underset{h(x)}{x^2} \qquad \text{for all } x$$

$$\lim_{x \to 0} (-x^2) = 0 \qquad \boxed{\lim_{x \to 0} x^2 \sin\left(\tfrac{1}{x}\right) = 0} \qquad \lim_{x \to 0} x^2 = 0$$

§1.8 – Continuity

Defn. In algebra a function f is <u>continuous</u> at a point $x=a$ if we can draw its graph near $x=a$ without lifting our pen(cil).

<u>NO</u>: jumps, holes, asymptotes

Defn. A function f is <u>continuous</u> at the point $x=a$ iff

$$\lim_{x\to a} f(x) = f(a)$$

$$\lim_{x\to a} f(x) = f\left(\lim_{x\to a} x\right) = f(a)$$

if and only if f is continuous at $x=a$!

Q. What kinds of functions are continuous on their <u>domains</u>?

- Polynomials
- Rational
- Trig function
- Roots
- absolute value
- logs
- exponential functions