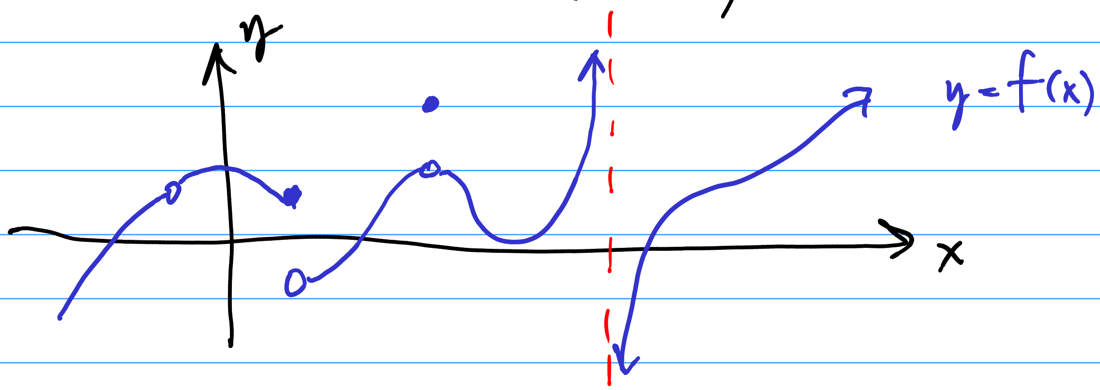
M242 8/24

§1.8 – Continuity

Def'n. A function f is <u>continuous</u> at $x=a$ iff

$$\lim_{x\to a} f(x) = f\left(\lim_{x\to a} x\right) = f(a)$$



holes: $f(a)$ = undef. but $\lim_{x\to a} f(x)$ does exist.

jumps: $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$, but they both exist.

asymptote: $\lim_{x\to a^-} f(x) = \pm\infty$ and/or $\lim_{x\to a^+} f(x) = \pm\infty$

Holes are not the worst. They are referred to as <u>removable discontinuities</u>.

We can define a new function F such that

$$F(x) = \begin{cases} f(x) & x \neq a \\ \lim_{x\to a} f(x) & x = a \end{cases}$$

Ex. $f(x) = \dfrac{x^2 - x - 2}{x-2}$ discontinuous at $x=2$

$$\lim_{x\to 2^-} \frac{x^2-x-2}{x-2} = \lim_{x\to 2^-} \frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} = \lim_{x\to 2^-} x+1 = 3$$

$$\lim_{x\to 2^+} \cdots = 3$$

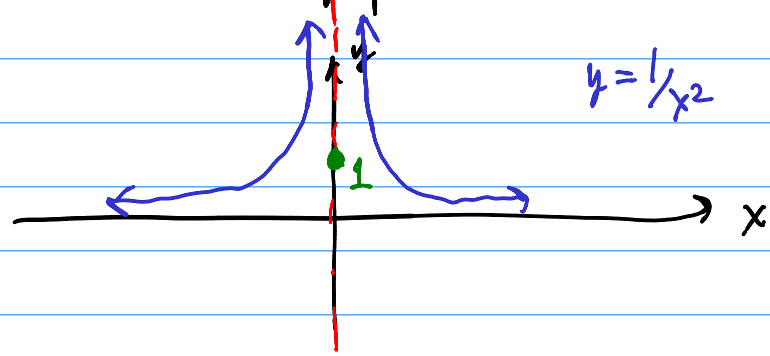
since these are equal, there is a hole at $x=2$.

$$F(x) = \begin{cases} f(x) & x \neq 2 \\ 3 & x = 2 \end{cases} = \begin{cases} \frac{x^2-x-2}{x-2} & x \neq 2 \\ 3 & x = 2 \end{cases}$$

or $F(x) = x+1$

Ex. $f(x) = \begin{cases} 1/x^2 & x \neq 0 \\ 1 & x = 0 \end{cases}$ Is f continuous?

$\lim_{x \to 0} \frac{1}{x^2} = +\infty$ so this function has a vertical asymptote at $x=0$.



Ex. $f(x) = 1 - \sqrt{1-x^2}$

dom(f): $1 - x^2 \geq 0$

$1 \geq x^2$

or $\sqrt{x^2} \leq \sqrt{1}$

$|x| \leq 1$

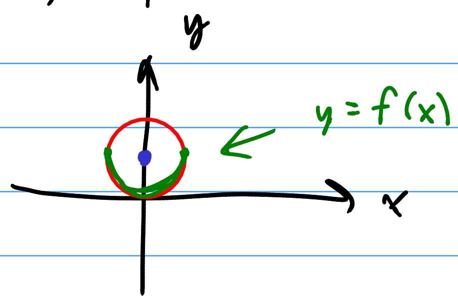
$\rightarrow -1 \leq x \leq 1$

$y = 1 - \sqrt{1-x^2}$

$(y-1)^2 = (-\sqrt{1-x^2})^2$!

$(y-1)^2 = 1 - x^2$

$x^2 + (y-1)^2 = 1$ circle! w/ center $(0,1)$ and radius 1



continuous on its entire domain.

Be careful w/ the endpoints!

$x=1$: $\lim_{x\to 1^-} f(x)$ only b/c we must stay in the domain

$$\lim_{x\to 1^-}\left(1-\sqrt{1-x^2}\right) = 1-\sqrt{0^+} = 1-0 = 1 \quad ✓$$

Ex. $f(x) = \dfrac{\ln(x) + \tan^{-1}(x)}{x^2-1}$ where is f continuous?

A. Find the domain.

$\ln(x)$ has domain: $(0,\infty)$