§1.7 Precise Definition of the Limit

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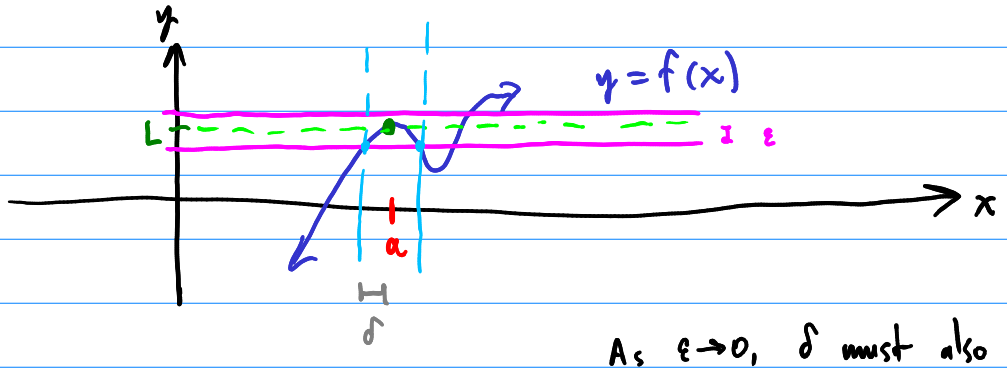
Defn. Let f be a function defined on an open interval containing $x=a$, but not necessarily at $x=a$. We say that **the limit of f as x approaches a is L** iff for any $\varepsilon>0$ we can find a $\delta=\delta(\varepsilon)$ such that whenever $|x-a|<\delta$, then $|f(x)-L|<\varepsilon$.



As $\varepsilon \to 0$, δ must also $\to 0$.

Given a limit $\lim_{x\to a} f(x) = L$, our job is to determine δ as a function of ε to satisfy the defn.

Example. $\lim_{x\to 3}(4x-5)=7$

Let $\varepsilon>0$ and assume $|x-3|<\delta$. Put $\varepsilon=4\delta$ so that

Then $|f(x)-L| = |4x-5-7|$

$= |4x-12|$ $\qquad \delta(\varepsilon) = \varepsilon/4$.

$= |4(x-3)|$

$= 4|x-3|$

$< 4\delta$

Example. $\lim_{x\to 1} 3x+2 = 5$

Let $\varepsilon>0$ and suppose $|x-1|<\delta$, Put $\delta = \frac{\varepsilon}{3}$.

then

$|f(x)-L| = |3x+2-5|$

$= |3x-3| = 3|x-1| < 3\delta = \varepsilon$