§1.7

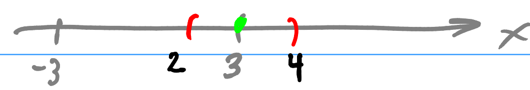
Ex. $\lim_{x\to 0^+}\sqrt{x} = 0$

Let $\varepsilon>0$, and suppose $|x-0|<\delta$ or $0<x<\delta$.

Then $|\sqrt{x}-0| = \sqrt{x} < \underbrace{\sqrt{\delta} = \varepsilon}$

then $\delta(\varepsilon) = \varepsilon^2$.

Ex. $\lim_{x\to 3} x^2 = 9$



Let $\varepsilon>0$ and suppose $|x-3|<\delta$.

Then,

$$|x^2-9| = |(x-3)(x+3)| = |x-3|\,\underset{?}{|x+3|}$$

Idea: Can we bound $|x+3|$ by some constant?

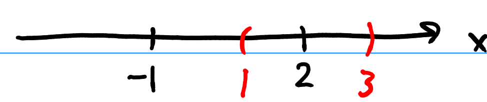
If $|x-3|<1$, then $|x+3|<7$

and $|x^2-9| = |x-3|\,|x+3| < 7\delta = \varepsilon$ and $\delta = \frac{\varepsilon}{7}$

Put $\boxed{\delta = \min\{1, \frac{\varepsilon}{7}\}.}$

Ex. $\lim_{x\to 2}(x^2-x-2) = 0$

$|x-2|<1 \Rightarrow |x+1|<4$

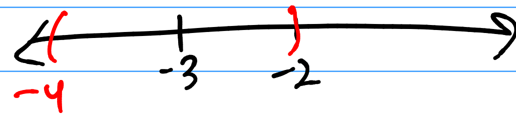


Let $\varepsilon>0$, suppose $|x-2|<\delta$.

Then $|x^2-x-2-0| = |x-2|\,\underset{?}{|x+1|} < 4\delta = \varepsilon \rightarrow \delta = \frac{\varepsilon}{4}$

Put $\delta = \min\{1, \varepsilon/4\}$.

Ex. $\lim_{x \to -3} x^2+5x = -6$



let $\varepsilon > 0$, $|x+3| < \delta$

$|x+3| < 1$

$|x+2| < 2$

$$|x^2+5x-(-6)| = |x^2+5x+6| = |x+2|\,|x+3| < 2\delta = \varepsilon$$

?

so $\delta = \min\{1, \varepsilon/2\}$.