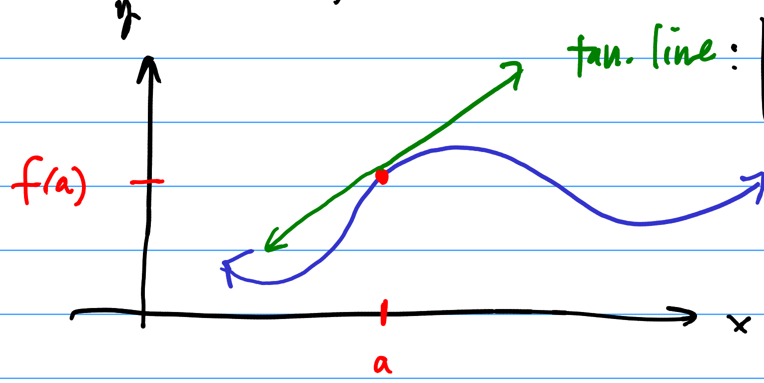
M242

§2.1 – Derivatives

Let f be a function, and $x=a$ be in the domain of f.

tan. line: $\boxed{y = f(a) + m(x-a)}$



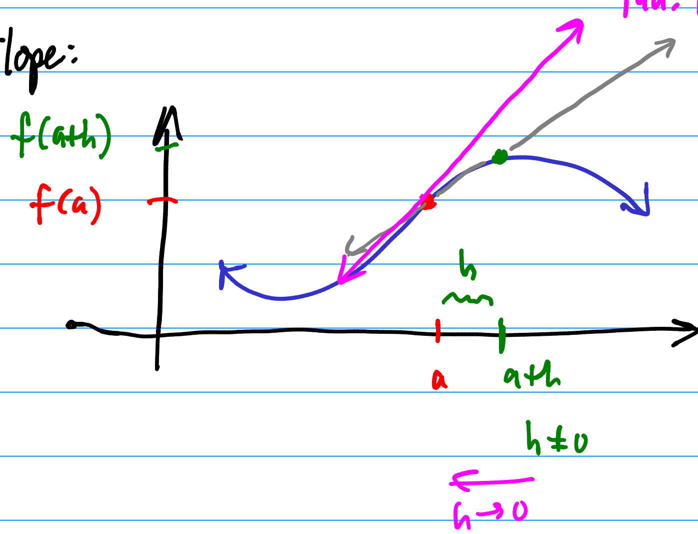
To know the equation of the tangent line, we must find its slope.

$$y = y_0 + m(x - x_0)$$

$\begin{cases} (x_0, y_0) & \text{any pt on } \ell \\ m & \text{is slope} \end{cases}$

tan. line: slope $= \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

Finding slope:



$$m = \frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{a+h-a}$$

$$m = \frac{f(a+h)-f(a)}{h}$$

Def'n. Let f be a function defined at $x=a$. The <u>derivative</u> of f at a is the number

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

provided the limit exists.

The equation of the tangent line to the graph of f at $x=a$ is

$$y = f(a) + f'(a)(x-a) \quad \circledast$$

Ex. $f(x) = x^2$ $\quad x = 2$
Find $f'(2)$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

1. $f(2+h) = (2+h)^2 = (2+h)(2+h) = 4 + 4h + h^2$

   $f(2) = 2^2 = 4$

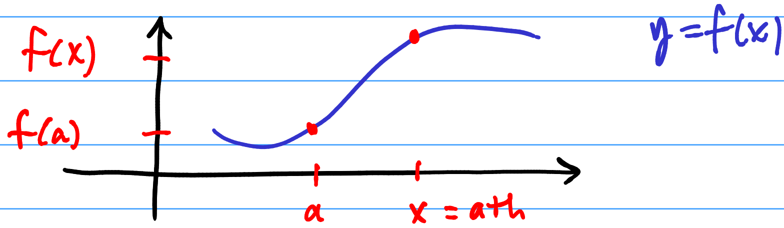
2. $f(2+h) - f(2) = \cancel{4} + 4h + h^2 - \cancel{4} = 4h + h^2$

3. $\dfrac{f(2+h) - f(2)}{h} = \dfrac{4h + h^2}{h} = \dfrac{h(4+h)}{h}$

4. $\displaystyle\lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = \lim_{h\to 0} 4+h = 4+0 = 4.$

so, $f'(2) = 4$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \underbrace{\lim_{x\to a} \frac{f(x) - f(a)}{x-a}} = \lim_{x\to a} \frac{\Delta y}{\Delta x} = \left.\frac{dy}{dx}\right|_{x=a}$$



<u>Newton's Notation:</u> $\quad \dot{f}(a)$

Ex. $f(x) = \frac{1}{x}$ $\qquad f'(2) = ?$ $\qquad f'(2) = \displaystyle\lim_{x\to 2} \frac{\frac{2}{2}\frac{1}{x} - \frac{1}{2}\frac{x}{x}}{x-2} = \rightarrow$

$$\rightarrow = \lim_{x\to 2} \frac{\frac{2-x}{2x}}{\frac{x-2}{1}} = \lim_{x\to 2}\left(\frac{2-x}{2x} \cdot \frac{1}{x-2}\right) = \lim_{x\to 2}\left(\frac{-\cancel{(x-2)}}{2x} \cdot \frac{1}{\cancel{x-2}}\right)$$

$$= \lim_{x\to 2} \frac{-1}{2x} = \frac{-1}{2\cdot 2} = \boxed{\frac{-1}{4} = f'(2)}$$

Ex. $f(x)=\sqrt{x}$ $\qquad f'(a) = ?$ $\qquad 0<a$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(\sqrt{a+h}-\sqrt{a})}{h} \frac{(\sqrt{a+h}+\sqrt{a})}{(\sqrt{a+h}+\sqrt{a})}$$

$$= \lim_{h\to 0} \frac{\cancel{a}+\cancel{h}-\cancel{a}\ \ 1}{\cancel{h}(\sqrt{a+h}+\sqrt{a})} = \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

$$= \frac{1}{\sqrt{a+0}+\sqrt{a}} = \frac{1}{2\sqrt{a}}.$$

$f(x)=\sqrt{x}$

$f'(a) = \frac{1}{2\sqrt{a}}, \quad a>0$

Ex. $f(x)=x^4$ $\qquad a=2$ $\qquad$ Find $f'(2)$.

$$f'(2) = \lim_{h\to 0} \frac{(2+h)^4 - 2^4}{h} \quad \text{or} \quad f'(2) = \lim_{x\to 2} \frac{x^4-2^4}{x-2}$$

Pascal's Δ:

$$(2+h)^4 = 1\cdot 2^4 + 4\cdot 2^3 h + 6\cdot 2^2 h^2 + 4\cdot 2 h^3 + 1 h^4$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$\frac{x^4-16}{x-2}$$

2	1	0	0	0	−16
	↓	2	4	8	16
	1	2	4	8	0