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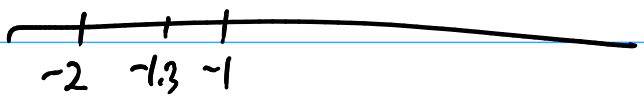
Ex. $f(x) = [\![x]\!]$

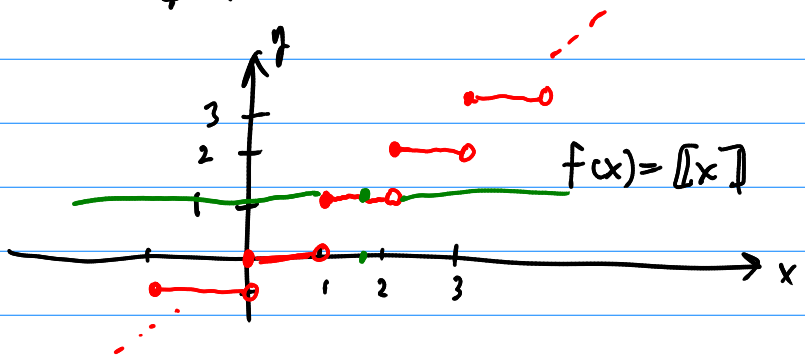


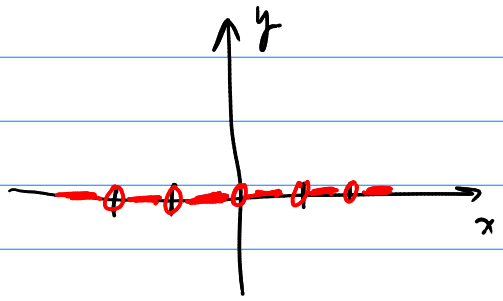
$[\![2.5]\!] = 2$ $\quad [\![-1.3]\!] = -2$

$[\![\pi]\!] = 3$

$[\![e]\!] = 2$

$$f'(x) = \begin{cases} 0 & \text{if } x \notin \mathbb{Z} \\ \text{undef.} & \text{if } x \in \mathbb{Z} \end{cases}$$





§ 2.5 – Chain Rule

Defn. Let f and g be differentiable functions such that

$$f'(g(a)) \quad \text{and} \quad g'(a) \quad \text{exist.}$$

Then, $$(f \circ g)'(x) = \Big(f(g(x))\Big)' = f'(g(x)) \cdot g'(x)$$

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let $f = f(u)$ and $u = u(x)$.

If $y = f(u)$, then $y' = f'(u) \cdot u'(x)$

or $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex. $y = (3x^2 + x)^5$

$f = u^5$ $\quad u = 3x^2 + x$

$f' = 5u^4$ $\quad u' = 6x + 1$

$y' = 5u^4(6x+1) = y' = 5(3x^2+x)^4(6x+1)$

Ex. $y = \cos^2 x = (\cos x)^2$

$f = u^2 \qquad u = \cos x$

$f' = 2u \qquad u' = -\sin x$

$y' = f'u' = 2u(-\sin x)$

$$\boxed{y' = -2\cos x \sin x}$$

Ex. $y = \cos(2x)$

$f = \cos u \qquad u = 2x$

$f' = -\sin u \qquad u' = 2$

$y' = (-\sin u)\cdot 2$

$$\boxed{y' = -2\sin(2x)}$$

Ex. $g(t) = \left(\frac{t-2}{2t+1}\right)^9$

$f = u^9 \qquad u = \frac{t-2}{2t+1}$

$f' = 9u^8 \qquad u' = 5/(2t+1)^2$

$$\left(\frac{t-2}{2t+1}\right)' = \frac{(2t+1)(t-2)' - (t-2)(2t+1)'}{(2t+1)^2} = \frac{2t+1-(2t-4)}{(2t+1)^2} = \frac{5}{(2t+1)^2}$$

$$g'(t) = 9\left(\frac{t-2}{2t+1}\right)^8 \frac{5}{(2t+1)^2} = \frac{45(t-2)^8}{(2t+1)^{10}}$$

Ex. $F(x) = \sin(\cos(\tan(x)))$

$F = f(u(N))$

$F' = f'u'N'$

$$\frac{dF}{dx} = \frac{df}{du}\frac{du}{dN}\frac{dN}{dx}$$

$f = \sin(u) \qquad u = \cos(N) \qquad N = \tan(x)$

$f' = \cos(u) \qquad u' = -\sin(N) \qquad N' = \sec^2(x)$

$$F'(x) = \cos(u)\left(-\sin(N)\right)\sec^2(x)$$

$$F'(x) = -\cos\left(\cos(\tan(x))\right)\sin\left(\tan(x)\right)\sec^2(x)$$

Ex. $y = \sqrt{\sec(x^3)}$

$f = \sqrt{u}$ $\quad u = \sec(N)$ $\quad N = x^3$

$f' = \frac{1}{2\sqrt{u}}$ $\quad u' = \sec(N)\tan(N)$ $\quad N' = 3x^2$

$$y' = \frac{3x^2\sec(N)\tan(N)}{2\sqrt{u}} = \frac{3x^2\sec(x^3)\tan(x^3)}{2\sqrt{\sec(x^3)}} = y'$$