M242 – §2.1–2.5 Review 9/10/18

A. Use the limit def'n to compute the derivatives:

1. $f(x)=x^4$
2. $f(x)=1/x$
3. $g(x)=\sqrt{x}$

B. Find an equation of the tan. line at $(a, f(a))$

4. $f(x)=x\cos x$, $a=\pi/2$
5. $f(x)=\frac{1}{(x-1)^2}$, $a=2$

C. Compute the derivatives of the functions

6. $f(x)=\sqrt{\tan x}$
7. $y=\sin x\cos x$
8. $y=\frac{\tan x}{x}$
9. $h(x)=|x|$ $(=\sqrt{x^2})$
10. $k(x)=\sin\left(\frac{1}{2}x^2\right)$
11. $g(t)=\sec(t)+\tan(\sqrt{t})$
12. $y=\sin^2 x-2\sin x+1$
13. $u=\frac{1}{\cos^2 x}+2x$
14. $y=\sqrt{1-\cos^2 x}$
15. $g(x)=\cos^2\left(\frac{1}{x}\right)$

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1. $f'(a)=\lim_{x\to a}\frac{x^4-a^4}{x-a}=\lim_{x\to a}(x^3+ax^2+a^2x+a^3)=a^3+a^3+a^3+a^3=4a^3 \Rightarrow f'(x)=4x^3$.

OR

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}=\lim_{h\to 0}\frac{\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4-\cancel{x^4}}{h}=\lim_{h\to 0}\frac{\cancel{h}(4x^3+6x^2h+4xh^2+h^3)}{\cancel{h}}$$

$$=\lim_{h\to 0}(4x^3+6x^2h+4xh^2+h^3)=4x^3.$$

2. $f'(a)=\lim_{x\to a}\frac{\frac{1}{x}-\frac{1}{a}}{x-a}=\lim_{x\to a}\frac{\frac{(a-x)}{xa}\cdot\frac{1}{x-a}}{\frac{x-a}{1}\cdot\frac{1}{x-a}}=\lim_{x\to a}\frac{-1}{xa}=\frac{-1}{a^2} \Rightarrow f'(x)=-1/x^2$

OR

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\lim_{h\to 0}\frac{\frac{(x-(x+h))}{x(x+h)}}{h}=\lim_{h\to 0}\frac{-\cancel{h}}{\cancel{h}x(x+h)}=\lim_{h\to 0}\frac{-1}{x(x+h)}=\frac{-1}{x^2}$$

3. $f'(a)=\lim_{x\to a}\frac{(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{x}+\sqrt{a})}=\lim_{x\to a}\frac{\cancel{x-a}\ 1}{\cancel{(x-a)}(\sqrt{x}+\sqrt{a})}=\lim_{x\to a}\frac{1}{\sqrt{x}+\sqrt{a}}=\frac{1}{2\sqrt{a}} \Rightarrow f'(x)=\frac{1}{2\sqrt{x}}$

OR

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{\cancel{x}+\cancel{h}-\cancel{x}\ 1}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

4. $f(x)=x\cos x$ $\quad f(\pi/2)=\pi/2\cdot 0=0.$

$f'(x)=\cos x-x\sin x$ $\quad f'(\pi/2)=0-\pi/2(1)=-\pi/2$

tan. line: $y=f(\pi/2)+f'(\pi/2)(x-\pi/2)=0+(-\pi/2)(x-\pi/2)=\boxed{-\pi/2\, x+\pi^2/4=y}$

5. $f(x)=\frac{1}{(x-1)^2}$ $\quad f(2)=\frac{1}{(2-1)^2}=1$ $\quad y=1+(-2)(x-2)=1-2x+4$

$f'(x)=\frac{-2}{(x-1)^3}$ $\quad f'(2)=-2$ $\quad \boxed{y=-2x+5}$

6. $f(x) = \sqrt{\tan x}$ $\quad f'(x) = \dfrac{\sec^2 x}{2\sqrt{\tan x}}$

7. $f(x) = \sin x \cos x$ $\quad f'(x) = (\cos x)(\cos x) + (\sin x)(-\sin x) = \cos^2 x - \sin^2 x = \cos(2x)$

or $\quad f(x) = \sin x \cos x = \frac{1}{2}\sin(2x)$

$\Longrightarrow f'(x) = \frac{1}{2}\cos(2x) \cdot 2 = \cos(2x)$

8. $y = \dfrac{\tan x}{x}$, $\quad y' = \dfrac{x\sec^2 x - \tan x}{x^2}$

9. $h(x) = \sqrt{x^2}$, $\quad h'(x) = \dfrac{2x}{2\sqrt{x^2}} = \dfrac{x}{\sqrt{x^2}} = \dfrac{x}{|x|}$.

10. $k(x) = \sin\left(\frac{1}{2}x^2\right)$, $\quad k'(x) = x\cos\left(\frac{1}{2}x^2\right)$

11. $g(t) = \sec(t) + \tan(\sqrt{t})$, $\quad g'(t) = \sec(t)\tan(t) + \dfrac{\sec^2(\sqrt{t})}{2\sqrt{t}}$

12. $y = \sin^2 x - 2\sin x + 1$, $\quad y' = 2\sin x \cos x - 2\cos x = 2\cos x(\sin x - 1)$

or

$y = (\sin x - 1)^2$, $\quad y' = 2(\sin x - 1) \cdot \cos x$

13. $u(x) = \dfrac{1}{\cos^2 x} + 2x = \sec^2 x + 2x$, $\quad u'(x) = 2\sec x \cdot \sec x \tan x + 2$

$= 2\sec^2 x \tan x + 2.$

14. $y = \sqrt{1 - \cos^2 x}$, $\quad y' = \dfrac{2\cos x \sin x}{2\sqrt{1 - \cos^2 x}} = \dfrac{\cos x \sin x}{\sqrt{1 - \cos^2 x}} = \dfrac{\cos x \sin x}{\sqrt{\sin^2 x}} = \dfrac{\cos x \sin x}{|\sin x|}$

OR

$y = \sqrt{(\sin x)^2}$ and use #9: $\quad y' = \dfrac{(\sin x)' \cdot \sin x}{|\sin x|} = \dfrac{\cos x \sin x}{|\sin x|}$

15. $g(x) = \cos^2\left(\frac{1}{x}\right)$, $\quad g'(x) = -2\cos(1/x)\sin(1/x) \cdot -1/x^2 = \dfrac{-\sin(2/x)}{x^2}$