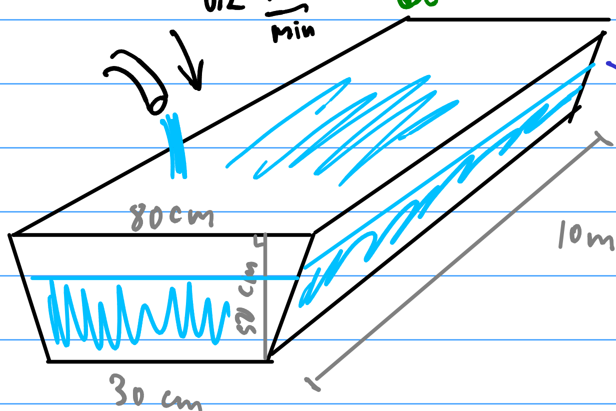
M242 - Calc I

9/19/18

2.8.10

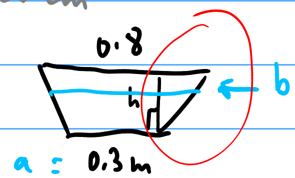
$0.2 \frac{m^3}{min} = \frac{dV}{dt}$



$h = 30\text{ cm}$ $\uparrow h$ Find $\frac{dh}{dt}$

Need: $V = ?$

$V = A \cdot l = 10A$



Need $b = b(h)$.

Known: when $h=0$ $b = .30$ m

when $h = 0.5$ $b = .8$ m

$$A = \frac{1}{2}(a+b)\cdot h$$
$$= \frac{1}{2}(0.3 + h + 0.3)h$$
$$= 0.3h + \frac{1}{2}h^2$$

$y - y_0 = m(x - x_0)$

$\boxed{\begin{array}{l} h \approx x \\ b \approx y \end{array}}$

$b - .3 = 1(h - 0)$ $m = \frac{\Delta b}{\Delta h} = \frac{.5}{.5} = 1$

$V = 10A = 3h + 5h^2$

$\boxed{b = h + 0.3}$ $\boxed{a = 0.3}$

Now take $\frac{d}{dt}\left[V = 3h + 5h^2\right]$

$$\frac{dV}{dt} = 3\frac{dh}{dt} + 10h\frac{dh}{dt} = \frac{dh}{dt}(3 + 10h)$$

$$\frac{dh}{dt} = \frac{dV}{dt}\cdot\left(\frac{1}{3+10h}\right) = 0.2\left(\frac{1}{3+10(.3)}\right) = \frac{2}{10}\cdot\frac{1}{6} = \frac{1}{30}\ m/min$$

2.3.16 $f(x) = \frac{x}{\left(x + d/x\right)} \cdot \frac{x}{x}$ $f'(x) = ?$

$f(x) = \frac{x^2}{x^2 + d}$, $x \neq 0$

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2} = \frac{2x(x^2+d) - 2x^3}{(x^2+d)^2} = \boxed{\frac{2xd}{(x^2+d)^2}}, \; x \neq 0.$$

$u = x^2$ $v = x^2 + d$

$u' = 2x$ $v' = 2x$

2.3.22 $\quad f(x) = \frac{1}{1+x^2}$ Find tan line at $(-1, \frac{1}{2})$

$$y = f(a) + \underbrace{f'(a)}_{\text{TBD}}(x-a) = \frac{1}{2} + \frac{1}{2}(x-(-1))$$

$$= \frac{1}{2} + \frac{1}{2}x + \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}x + 1}$$

$$f(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$$

$f = u^{-1}$ $u = 1 + x^2$

$f' = -u^{-2}$ $u' = 2x$

$$y' = f'u' = -u^{-2} \cdot 2x = \frac{-2x}{(1+x^2)^2}$$

$$f'(-1) = \frac{-2(-1)}{(1+(-1)^2)^2} = \frac{2}{4} = \frac{1}{2}$$

2.3.24: $\quad f(x) = y = \frac{\sqrt{x}}{4+x}$ $(1, \frac{1}{5})$

T: $\quad y = f(a) + f'(a)(x-a)$

N: $\quad y = f(a) - \frac{1}{f'(a)}(x-a)$

$$f'(x) = \frac{(4+x)\frac{1}{2\sqrt{x}} - \sqrt{x}}{(4+x)^2}$$

$$f'(1) = \frac{(4+1)(\frac{1}{2}) - 1}{5^2} = \frac{5/2 - 1}{25}$$

$$= \boxed{\frac{3}{50}} = 0.06$$

m_T

$$m_N = \boxed{-\frac{50}{3}}$$

2.9.8 a)

$f(t) = y = \tan(\sqrt{3t})$ Find $dy = f'(t)\,dt$

$f'(t) =$

$f = \tan(u) \quad u = \sqrt{N} \quad N = 3t$

$f' = \sec^2 u \quad u' = \frac{1}{2\sqrt{N}} \quad N' = 3$

$$\boxed{dy = 3\frac{1}{2\sqrt{3t}} \cdot \sec^2(\sqrt{3t})\ dt}$$

b) $y = \frac{1-v^2}{1+v^2}$ $y = \underbrace{f'(v)}\,dv$

$$L = f(a) + \overbrace{f'(a)(x-a)}^{dy}$$

$f(x+dx) \approx f(a) + dy$