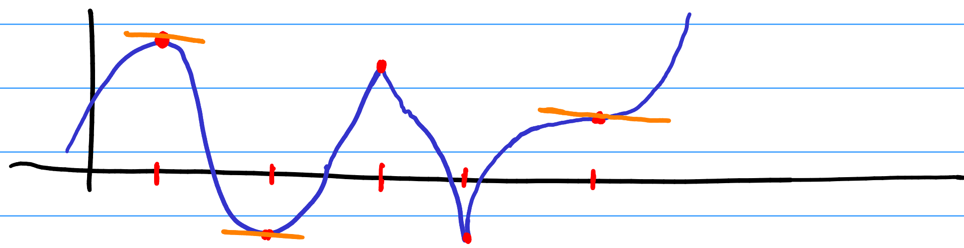
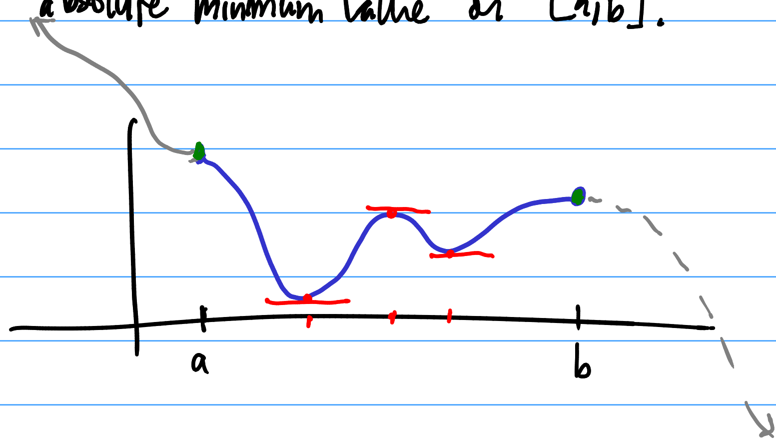
M242 — 9/21/18

§3.1 – Extreme Values

Critical number: $f'(c) = \begin{cases} 0 \\ \text{undef.} \end{cases}$ but c is in $\text{dom}(f)$.



Extreme Value Theorem. Let f be a continuous function on $[a,b]$. Then f attains an absolute maximum and absolute minimum value on $[a,b]$.



CN ~ CP

Endpoints

Ex. $f(x) = x^3 - 3x^2 + 1 \qquad -\frac{1}{2} \le x \le 4$

Find the absolute extrema of f on $-\frac{1}{2} \le x \le 4$.

Endpoints: $f(-\tfrac{1}{2}) = (-\tfrac{1}{2})^3 - 3(-\tfrac{1}{2})^2 + 1 = -\frac{1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$

$f(4) = 4^3 - 3 \cdot 4^2 + 1 = 64 - 48 + 1 = 17$ ← Abs. Max.

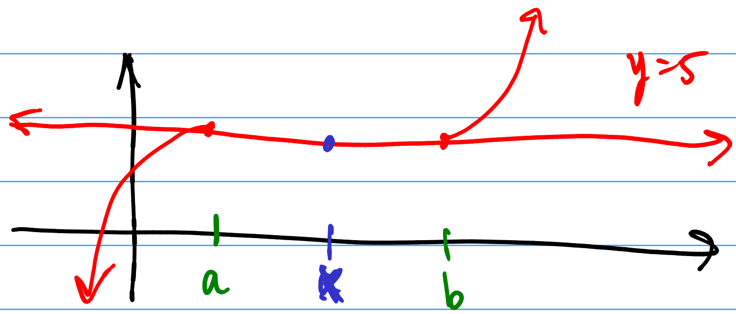
CP's: $f'(x) = 3x^2 - 6x = 0 \rightarrow 3x(x-2) = 0$

$x = 0, \quad x = 2$ ← Critical numbers

$f(0) = 0^3 - 3\cdot 0^2 + 1 = 1$

$f(2) = 2^3 - 3\cdot 2^2 + 1 = 8 - 12 + 1 = -3$ } ← critical values

Abs. Min.



5 = abs. max = abs. min.

Ex. $a, b > 0$ $\qquad f(x) = x^a (1-x)^b \qquad 0 \le x \le 1$

Find abs. extreme values of f on $0 \le x \le 1$.

Endpoints: $f(0) = 0^a (1-0)^b = 0 \cdot 1 = 0$

$f(1) = 1^a (1-1)^b = 1 \cdot 0 = 0$

CP's: $f'(x) = a x^{a-1}(1-x)^b - b x^a (1-x)^{b-1} = 0$

$$= x^{a-1}(1-x)^{b-1}\left[a(1-x) - bx\right] = 0$$

$x^{a-1} = 0$
$\rightarrow x = 0$

$(1-x)^{b-1} = 0$
$\rightarrow 1 - x = 0$
$\rightarrow x = 1$

$a(1-x) - bx = 0$
$a - ax - bx = 0$
$a = (a+b)x$
$0 < x = \frac{a}{a+b} < 1$ $\qquad$ We Care!

$$f\left(\frac{a}{a+b}\right) = \left(\frac{a}{a+b}\right)^a \left(1 - \frac{a}{a+b}\right)^b = \frac{a^a}{(a+b)^a} \cdot \frac{b^b}{(a+b)^b} = \frac{a^a b^b}{(a+b)^{a+b}}$$

So, $\frac{a^a b^b}{(a+b)^{a+b}} > 0$ and hence is the absolute max.

$a = 1 \quad b = x$

$$\frac{x^x}{(1+x)^{1+x}} = 1? \qquad x^x = (1+x)^{1+x}$$

$$x^x - (1+x)^{1+x} = 0$$

Apply Bisection Method to determine if this is possible.