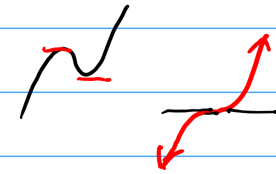
§3.1. # 18 :

Ex. $f(x) = 8x^3 + x^2 + 8x$ Find CN



$f'(x) = 24x^2 + 2x + 8 = 0$

$\cancel{2}(12x^2 + x + 4) = 0$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(12)(4)}}{2(12)} = \dfrac{-1 \pm \sqrt{-?}}{2(12)} = \underline{\text{complex}}.$

No CN's.

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§3.3 – Derivatives and the shapes of graph

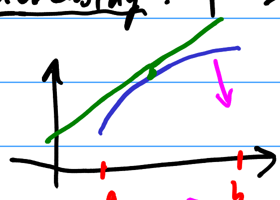
Def'n. A function f is said to be increasing at a $x=a$ iff $f'(a) > 0$.
And decreasing if $f'(a) < 0$.

Let (a,b) be an interval in the domain of f.

f is increasing on	(a,b)	iff	$f'(c) > 0$	for all c,	$a<c<b$.
" decreasing	"	"	$f'(c) < 0$	for all c,	$a<c<b$.
constant			$f'(c) = 0$	for all c,	$a<c<b$.

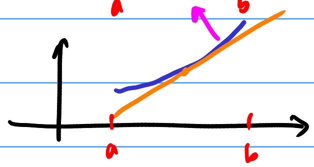
Increasing : $f' > 0$

Concave down



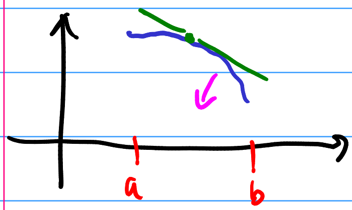
tan line is above the curve.
slope of tan. line is decreasing. } $f''(x) < 0$

Concave up

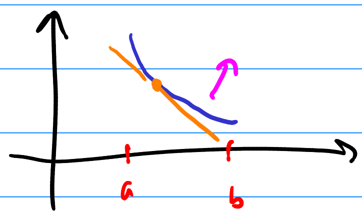


tan. line is below the curve
slope of tan. line is increasing } $f''(x) > 0$

Decreasing: $f'(x)<0$



$f''(x)<0$ concave down

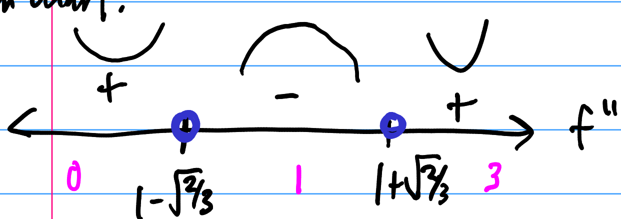


$f''(x)>0$ concave up

Ex. $f(x)=x^4-4x^3+2x^2-1$ Find the intervals of concavity.

$$f'(x)=4x^3-12x^2+4x$$
$$f''(x)=12x^2-24x+4=4(3x^2-6x+1)$$
$$=12\left((x^2-2x+1)+\tfrac{1}{3}-1\right)$$
$$=\cancel{12}\left[(x-1)^2-\tfrac{2}{3}\right]=0$$

sign chart:



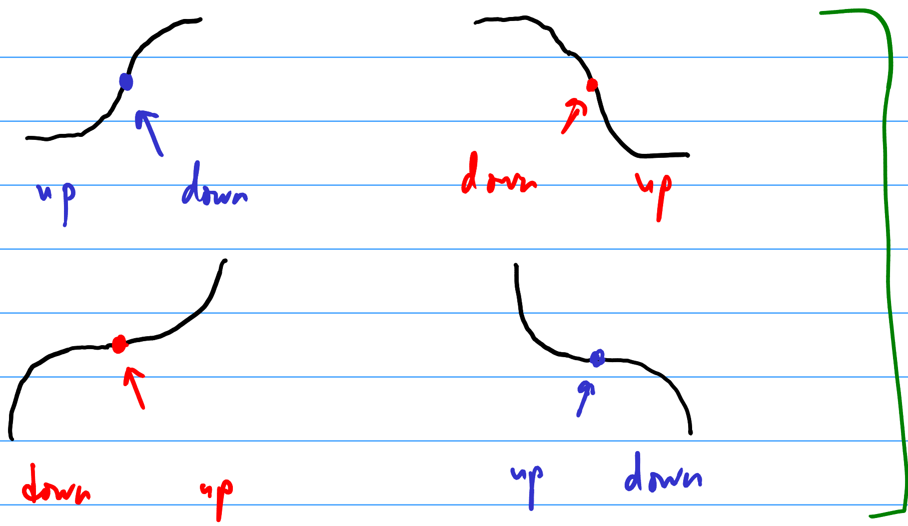
$$\sqrt{(x-1)^2}=\sqrt{2/3}$$
$$|x-1|=\sqrt{2/3}$$
$$x-1=\pm\sqrt{2/3}$$
$$\boxed{x=1\pm\sqrt{2/3}}$$

$f''(0)=4>0$
$f''(1)=-8<0$
$f''(3)=12(9)-24(3)+4=96-72+4=+$

Intervals: Concave up: $(-\infty,1-\sqrt{2/3})\cup(1+\sqrt{2/3},\infty)$
Concave down: $(1-\sqrt{2/3},1+\sqrt{2/3})$

Defn. Let f be a function whose 2^{nd} derivative is defined.
A point $(a, f(a))$ is called an inflection point iff $f''(a)=0$ and f changes concavity at this point.

black graph is $y=f(x)$



$f''(a)=0$

Ex. $f(x) = \sin(x)$. Find all inflection pts.

$f'(x) = \cos(x)$

$f''(x) = -\sin(x) = 0$

$-\sin(x) = 0 \Rightarrow \boxed{x = n\pi}, \; n \in \mathbb{Z}$

all inflection points.