M242 9/26/18

§3.3, 4, 5 - Sketching graphs

Want:
- Domain
- zeros of function (x-int)
- Critical pts
- Inflection pts
- vertical asymptotes?
- horizontal asymptotes - behavior at "$x = \pm\infty$"

Ex. $f(x) = x^4 - 2x^2 + 3$

- domain = $\mathbb{R}$
- continuous

zeros: $x^4 - 2x^2 + 3 = 0$ solve for x

$x^4 - 2x^2 + 1 = -3 + 1$

$(x^2-1)^2 = -2$ ← No real solutions.

Graph never crosses x-axis.

End behavior: ↑ ... ↗

y-int: $x=0$: $f(0) = 0^4 - 2\cdot 0^2 + 3 = 3$ $(0,3)$

Critical Points: $f'(x) = \begin{cases} 0 \\ \text{undef.} \end{cases}$

$f(x) = x^4 - 2x^2 + 3$

$f'(x) = 4x^3 - 4x$

$f'(x) = 0$: $4x^3 - 4x = 0$

$4x(x^2-1) = 0$

$4x(x+1)(x-1) = 0$

$x = 0, -1, 1$ CN's

$f(-1) = (-1)^4 - 2(-1)^2 + 3 = 2$

$f(1) = (1)^4 - 2(1)^2 + 3 = 2$

Critical pts: $(0,3)$, $(1,2)$, and $(-1,2)$

Inflection pts: $f''(x) = 0$ and changes sign.

$f'(x) = 4x^3 - 4x$

$\underbrace{f''(x) = 12x^2 - 4} \rightarrow 12x^2 - 4 = 0$

$12x^2 = 4$

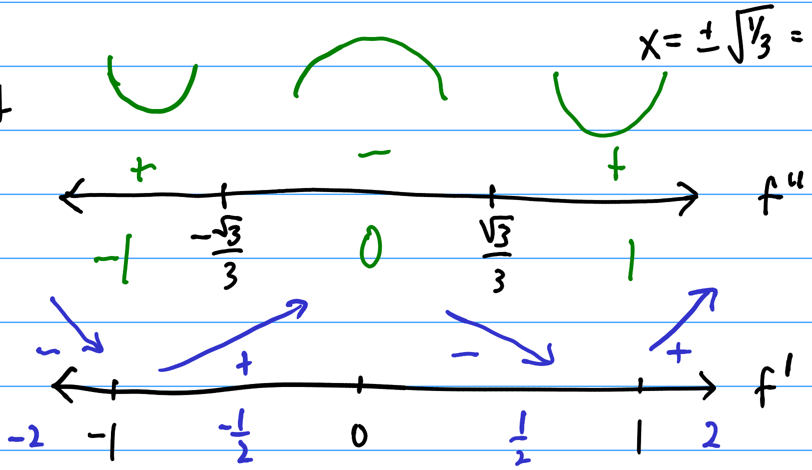
$x^2 = 4/12 = 1/3$

$x = \pm\sqrt{1/3} = \pm\frac{\sqrt{3}}{3}$

Sign chart

$f''(-1) = 12 - 4 = 8 > 0$

$f''(0) = -4 < 0$



$f'(x) = 4x(x+1)(x-1)$

$f'(-2) = - \cdot - \cdot - = -$

$f'(-1/2) = - \cdot + \cdot - = +$

$f'(1/2) = + \cdot + \cdot - = -$

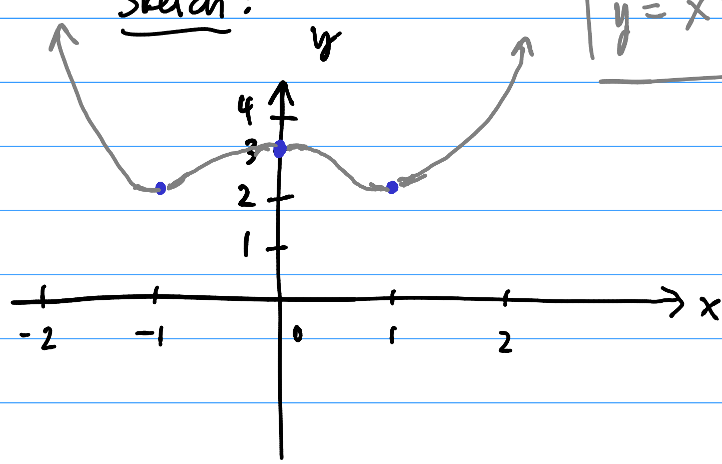
$f'(2) = + \cdot + \cdot + = +$

CP's: Local min: $(-1,2)$ and $(1,2)$

Local max: $(0,3)$

Sketch:

$\boxed{y = x^4 - 2x^2 + 3}$



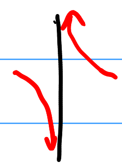
Ex. $f(x) = \dfrac{x^2}{x-1}$

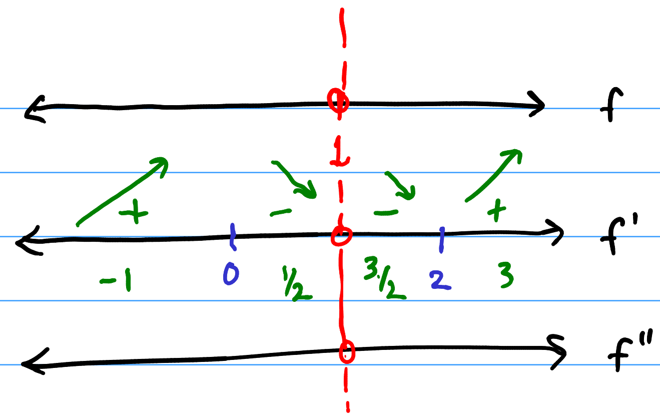
domain: $x - 1 \neq 0$ $\quad x \neq 1$

$$\lim_{x \to 1^-} \frac{x^2}{x-1} = \frac{(1^-)^2}{1^- - 1} = \frac{1}{0^-} = -\infty$$

$$\lim_{x \to 1^+} \frac{x^2}{x-1} = \frac{(1^+)^2}{1^+ - 1} = \frac{1}{0^+} = +\infty$$

Shape:





$$f'(x) = \frac{x(x-2)}{(x-1)^2} = \frac{x(x-2)}{+}$$

Consider: $$\lim_{x \to \infty} \frac{x^2 \cdot \frac{1}{x^2}}{(x-1) \cdot \frac{1}{x^2}} = \lim_{x \to \infty} \frac{1}{\frac{1}{x} - \frac{1}{x^2}} = \frac{1}{0-0} = \frac{1}{0} = \infty$$

$\dfrac{x^2}{x-1}$

$$\begin{array}{c|ccc} 1 & 1 & 0 & 0 \\ & & 1 & 1 \\ \hline & 1 & 1 & \boxed{1} \end{array}$$

$$\frac{x^2}{x-1} = x + 1 + \underbrace{\frac{1}{x-1}}$$

As $x \to \infty$ for $x + 1 + \frac{1}{x-1}$, the graph of our function will approach the line $y = x + 1$.

So, $y = x + 1$ is a slant asymptote.

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x & y-intercepts $\qquad f(x) = \dfrac{x^2}{x-1}$

$x = 0$: $\quad f(0) = \dfrac{0^2}{0-1} = 0$, so $(0,0)$

$y = 0$: $\quad \dfrac{x^2}{x-1} = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0 \qquad (0,0)$

$\underline{CP's}$. $f'(x) = \begin{cases} 0 \\ \text{undef.} \end{cases}$

$$f'(x) = \left(\frac{x^2}{x-1}\right)' = \left(\frac{(x-1)(2x) - x^2(1)}{(x-1)^2}\right) = \frac{2x^2-2x-x^2}{(x-1)^2}$$

$$f'(x) = \frac{x^2-2x}{(x-1)^2}$$

CN: $\frac{x^2-2x}{(x-1)^2} = 0 \Rightarrow x^2-2x=0 \quad x(x-2)=0 \quad \boxed{x=0, \; x=2}$ $\quad f'(x)=0$

$(x-1)^2 = 0 \Rightarrow \cancel{x=1} \leftarrow$ $\underline{\text{not}}$ in domain.

0: max. $\quad (0,0)$

2: min. $\quad f(2) = \frac{(x^2)}{x-1} = \frac{2^2}{2-1} = \frac{4}{1} = 4$

$(2,4)$

$\boxed{RE}$ f''

$\underline{\text{Sketch}}$: