M242 9/27/18

§ 3.3–5 Curve Sketching

Ex. $f(x) = \dfrac{x^2+5x}{25-x^2}$

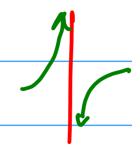
domain: $25 - x^2 \neq 0 \Rightarrow \underline{x \neq 5, -5}$

zeros: $x^2 + 5x = 0 \Rightarrow x(x+5) = 0$ $\boxed{x = 0}$ $x = \cancel{-5}$

$\boxed{(x,y) = (0,0)}$ y-int

VA

$$\lim_{x\to 5^-} \frac{x^2+5x}{25-x^2} = \lim_{x\to 5^-} \frac{x\cancel{(x+5)}}{(5-x)\cancel{(5+x)}} = \frac{5^-}{5-5^-} = \frac{5}{0^+} = +\infty$$



$x=5$

$$\lim_{x\to 5^+} \frac{x}{5-x} = \frac{5^+}{5-5^+} = \frac{5}{0^-} = -\infty$$

$$\lim_{x\to -5} \frac{x}{5-x} = \frac{-5}{5-(-5)} = \frac{-5}{10} = -\frac{1}{2}$$ HOLE! : $\boxed{(-5, -\tfrac{1}{2})}$ NOT on graph.

HA? $$\lim_{x\to\infty} \frac{(x^2+5x)\,{}^{1}\!/\!{}_{x^2}}{(25-x^2)\,{}^{1}\!/\!{}_{x^2}} = \lim_{x\to\infty} \frac{1 + 5/x}{\frac{25}{x^2} - 1} = \frac{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{5}{x}}{\lim_{x\to\infty} \frac{25}{x^2} - \lim_{x\to\infty} 1} = \frac{1+0}{0-1} = \frac{1}{-1} = -1$$

$\boxed{y = -1}$ Horizontal asymptote

$f(x) = \dfrac{x^2+5x}{25-x^2}$

$$25 - x^2 \overline{\big)\, x^2 + 5x + 0} \quad -1 + \frac{5x+25}{25-x^2}$$

$$-(x^2 + 0x - 25)$$

$$5x + 25$$

$$f(x) = -1 + \frac{-5(x+5)}{x^2-25} = -1 - \frac{5\cancel{(x+5)}}{(x-5)\cancel{(x+5)}}$$ $x \neq -5$

$$f(x) = -1 - \frac{5}{x-5}, \quad x \neq -5$$ $$\frac{-5}{x-5} = -5(x-5)^{-1}$$

$$f'(x) = 0 + \frac{5}{(x-5)^2}, \quad \underline{x \neq -5}$$

$\underline{CN's}$: $f'(x) = \begin{cases} 0 \\ \text{undef.} \end{cases}$

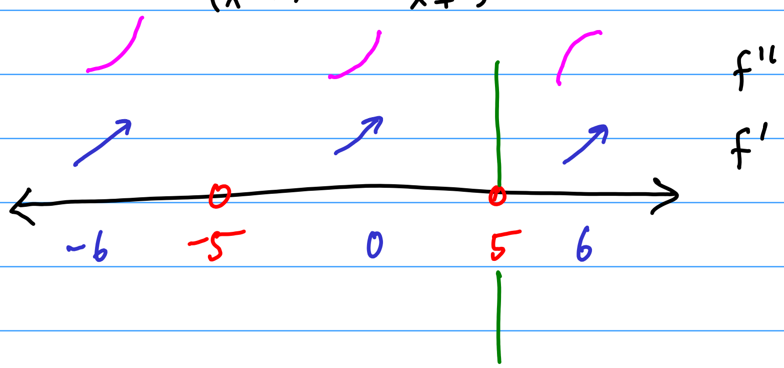
$f'(x) = 0?$: $5 \neq 0$

$f'(x) = \text{undef.}$ $(x-5)^2 = 0$, $x \neq 5$, $x \neq -5$

$$f'(x) = \frac{5}{(x-5)^2}, \quad x \neq -5 \quad \text{or} \quad f'(x) = 5(x-5)^{-2}$$

$$f''(x) = -10(x-5)^{-3} = \frac{-10}{(x-5)^3}, \quad x \neq -5, \; x \neq 5$$

$f''(x) = 0?$: $-10 \neq 0$



$$f'(x) = \frac{5}{(x-5)^2} > 0 \quad \text{always increasing}$$

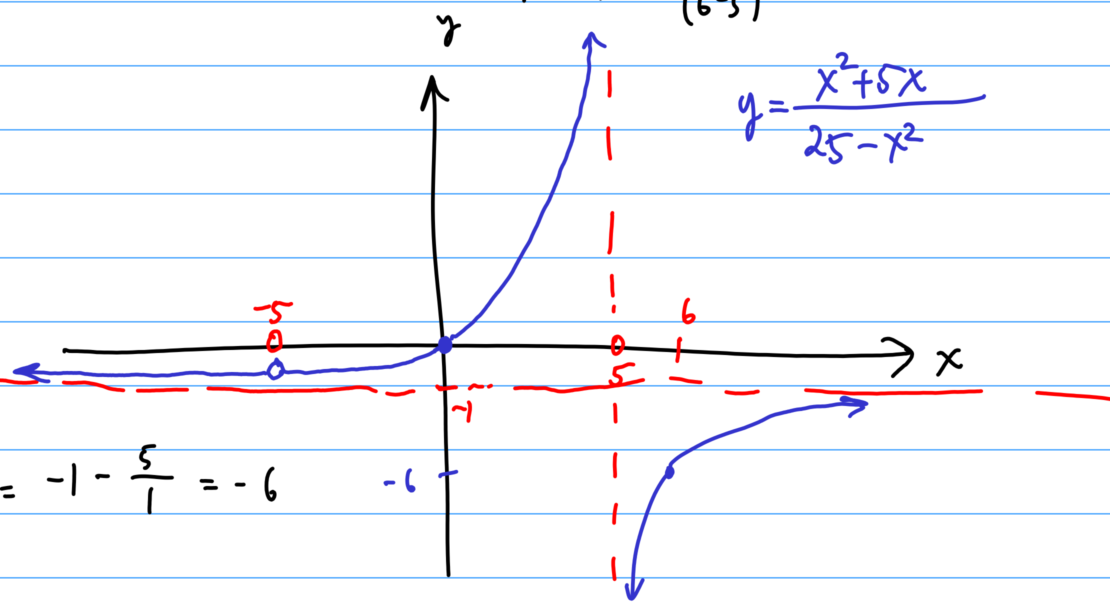
$$f''(x) = \frac{-10}{(x-5)^3} = \frac{-}{?}$$

$f''(-6) = \frac{-}{(-6-5)^3} = \frac{-}{-} = +$

$f''(0) = \frac{-}{-} = +$

$f''(6) = \frac{-}{(6-5)^3} = \frac{-}{+} = -$

$\underline{\text{Sketch}}$:

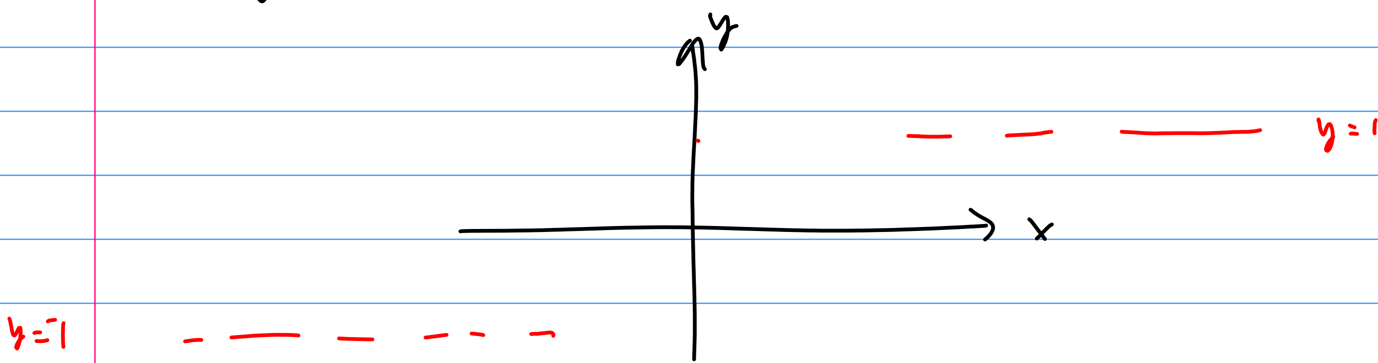


$$f(6) = -1 - \frac{5}{x-5} = -1 - \frac{5}{1} = -6$$

Ex. $f(x) = \dfrac{x+1}{\sqrt{x^2+1}}$

$$\lim_{x\to\infty} \frac{(x+1)\,\frac{1}{x}}{\left(\sqrt{x^2+1}\right)\frac{1}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}} = \frac{1+0}{\sqrt{1+0}} = \frac{1}{\sqrt{1}} = 1 \qquad y=1$$

$$\lim_{x\to-\infty} \frac{(x+1)\,\frac{1}{x}}{\sqrt{x^2+1}\;\frac{1}{-\sqrt{x^2}}} = \lim_{x\to\infty} \frac{1+\frac{1}{x}}{-\sqrt{1+\frac{1}{x^2}}} = \frac{1}{-\sqrt{1}} = -1 \qquad y=-1$$



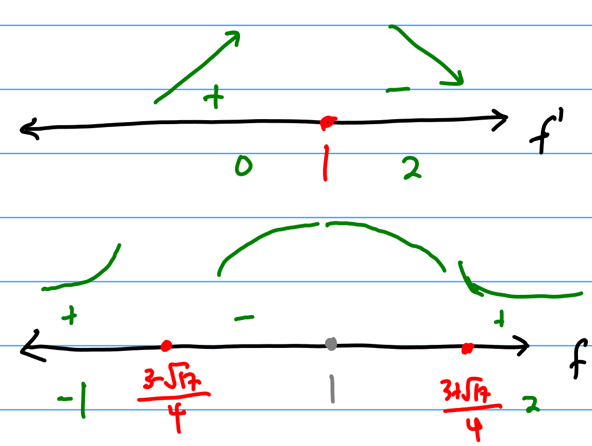
$$f(0) = \frac{0+1}{\sqrt{0^2+1}} = \frac{1}{1} = 1 \qquad \frac{x+1}{\sqrt{x^2+1}} = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1$$

so $(0,1)$ and $(-1,0)$ are the intercepts.

Domain $= \mathbb{R}$, no VAs. Continuous everywhere.

$$f'(x) = \frac{\sqrt{x^2+1}\,(1) - (x+1)\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}}{\left(\sqrt{x^2+1}\right)^2} = \frac{x^2+1-(x^2+x)}{\left(\sqrt{x^2+1}\right)^3} = \frac{-x+1}{(x^2+1)^{3/2}}$$

CP: $-x+1 = 0 \Rightarrow -x = -1 \Rightarrow \left(1, \frac{\sqrt{2}}{2}\right)$



$$f''(x) = \frac{\sqrt{x^2+1}^{\,3}(-1) - (-x+1)\,\frac{3}{\cancel{2}}(x^2+1)^{1/2}\cdot\cancel{2}x}{(x^2+1)^3}$$

$$= \frac{\sqrt{x^2+1}\left(-(x^2+1) - (-3x^2+3x)\right)}{(x^2+1)^3}$$

$$f''(x) = \frac{2x^2-3x-1}{(x^2+1)^{5/2}} = \frac{2\left(x-\frac{3+\sqrt{17}}{4}\right)\left(x-\frac{3-\sqrt{17}}{4}\right)}{(x^2+1)^{5/2}}$$

$f''(x)=0$: $2x^2-3x-1=0$

$$2\left(x^2-\tfrac{3}{2}x+\tfrac{9}{16}-\tfrac{1}{2}-\tfrac{9}{16}\right)=0$$

$$\left(x-\tfrac{3}{4}\right)^2=\tfrac{17}{16}$$

$$x=\tfrac{3}{4}\pm\tfrac{\sqrt{17}}{4}=\frac{3+\sqrt{17}}{4},\ \frac{3-\sqrt{17}}{4}$$

Putting it all together, sketch: