M242 - Calc I 10/3/18

§3.9 - Antiderivatives

Consider a function $y=f(x)$. An antiderivative of f is a function F satisfying $F'(x)=f(x)$ for all x in $\text{dom}(f)$.

Ex. $f(x)=x^2$

Guess: $F(x)=x^3$ $\qquad F'(x)=3x^2$ ✗

Instead: $\boxed{F(x)=\frac{1}{3}x^3}$ $\qquad F'(x)=\frac{1}{\cancel{3}}\cdot\cancel{3}x^2 = x^2 = f(x)$

This is an antiderivative of x^2.

The general antiderivative of $f(x)=x^2$ is the family of functions $\boxed{F(x)=\frac{1}{3}x^3+C}$ where C can be any number.

Ex. $h(x)=x^5$

$H(x)=\frac{1}{6}x^6+C$ $\qquad H'(x)=\frac{1}{\cancel{6}}\cancel{6}x^5+0 = x^5 = h(x)$ ✓

Power Rule for Antiderivatives:

if $f(x)=x^n$, then $F(x)=\frac{1}{n+1}x^{n+1}+C$, $n\neq -1$

Ex. $f(x)=\sqrt{x}=x^{1/2}$ $\qquad F(x)=\frac{2}{3}x^{3/2}+C$

Ex. $g'(x) = 4\sin x + \dfrac{2x^5 - \sqrt{x}}{x}$ Find g.

$g'(x) = 4\sin x + 2x^4 - x^{-1/2}$

$$\boxed{g(x) = -4\cos x + \frac{2}{5}x^5 - 2x^{1/2} + C}$$

Ex. $f'(x) = x\sqrt{x}$, $f(1) = 2$

$f'(x) = x \cdot x^{1/2} = x^{3/2}$

$f(x) = \frac{2}{5}x^{5/2} + C$

$f(1) = \frac{2}{5}1^{5/2} + C = 2$

$C = 2 - 2/5 = 8/5$

$$\boxed{f(x) = \frac{2}{5}x^{5/2} + 8/5}$$

Ex. $f''(x) = 12x^2 + 6x - 4$ $\rightarrow \begin{cases} f(0) = 4 \\ f(1) = 1 \end{cases}$

$f''(x) = (f'(x))'$

$f'(x) = \frac{12}{3}x^3 + \frac{6}{2}x^2 - 4x + C = \underbrace{4x^3 + 3x^2 - 4x + C}$

$f(x) = \frac{4}{4}x^4 + \frac{3}{3}x^3 - \frac{4}{2}x^2 + Cx + D = x^4 + x^3 - 2x^2 + \overset{-3}{\cancel{C}}x + \overset{4}{\cancel{D}}$

$f(0) = 0^4 + 0^3 - 2 0^2 + C \cdot 0 + D = D = 4$

$f(1) = 1^4 + 1^3 - 2 \cdot 1^2 + C \cdot 1 + 4 = \underbrace{1 + 1 - 2} + C + 4 = C + 4 = 1$

$C = -3$

$$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$$

Ex. $a(t) = 6t + 4$ acceleration

$$\begin{cases} s(0) = 9 \text{ cm} \\ v(0) = -6 \text{ cm/s} \end{cases}$$

we construct the position function $s(t)$.

$$v(t) = s'(t)$$
$$a(t) = v'(t) = s''(t)$$

$$v(t) = \frac{6t^2}{2} + 4t + C = 3t^2 + 4t + C$$

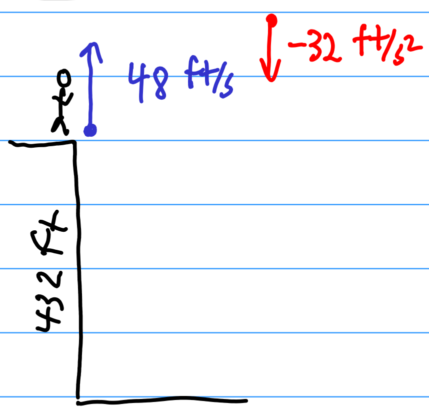
$$v(0) = C = -6$$

$$v(t) = 3t^2 + 4t - 6$$

$$s(t) = t^3 + 2t^2 - 6t + C$$
$$s(0) = C = 9$$

$$\boxed{s(t) = t^3 + 2t^2 - 6t + 9}$$

Ex.



Find $h(t)$

$$\begin{cases} h(0) = 432 \\ h'(0) = 48 \end{cases}$$

$$h''(t) = -32$$
$$h'(t) = -32t + C$$
$$h'(0) = \underbrace{0 + C = 48}$$
$$h'(t) = -32t + 48$$
$$h(t) = -16t^2 + 48t + C$$
$$h(0) = C = 432$$

$$\boxed{h(t) = -16t^2 + 48t + 432}$$

RE. Find Max height

Q. How long until it hits the ground?

$h(t) = 0$ solve for t.

$-16t^2 + 48t + 432 = 0$

$-16\underbrace{(t^2 - 3t - 27)} = 0$

$t^2 - 3t + 9/4 = 27 + 9/4$

$(t - 3/2)^2 = \frac{117}{4}$

$t = 3/2 \pm \frac{\sqrt{117}}{2}$ $t = \frac{3 + \sqrt{117}}{2}$ sec