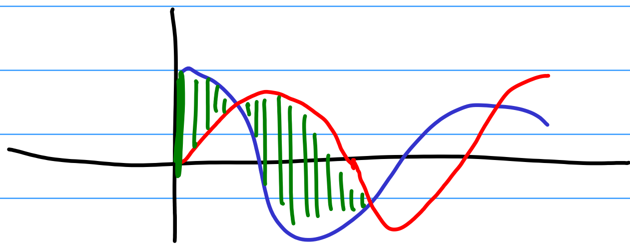
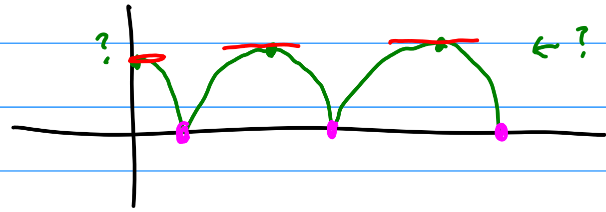
M242 10/4/18

Ch. 3 Applications of Derivatives

PP#1. Show that $0 \leq \underbrace{|\sin x - \cos x|} \leq \sqrt{2}$ for all x.





$$f(x) = |\sin x - \cos x| = \sqrt{(\sin x - \cos x)^2}$$

$$f'(x) = \frac{\cancel{2}(\sin x - \cos x)(\sin x + \cos x)}{\cancel{2}\sqrt{(\sin x - \cos x)^2}}$$

$f = \sqrt{u}$ $\quad u = N^2$ $\quad N = \sin x - \cos x$

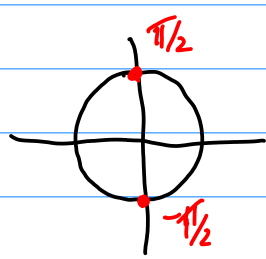
$f' = \frac{1}{2\sqrt{u}}$ $\quad \underbrace{u' = 2N \quad N' = \cos x + \sin x}$

$$f'(x) = \frac{\boxed{\sin^2 x - \cos^2 x}}{|\sin x - \cos x|} = \frac{\cancel{\tfrac{1}{2}} - \frac{1}{2}\cos(2x) - (\cancel{\tfrac{1}{2}} + \frac{1}{2}\cos(2x))}{|\sin x - \cos x|}$$

$$= \frac{-\cos(2x)}{|\sin x - \cos x|}$$

C.N.s: Top: $-\cos(2x) = 0$

$\cos\theta = 0$?

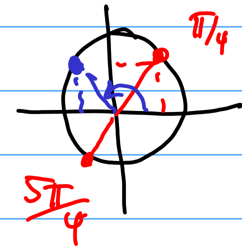


$2x = \theta = n\pi/2$ ← for $n = \pm 1, \pm 3, \pm 5, \ldots$

$= \pi/2 + 2\pi n$ for all n

and $-\pi/2 + 2\pi n$

then $\boxed{x = n\,\pi/4}$ $\quad n = \pm 1, \pm 3, \dots$



Both:

$$\sin x - \cos x = 0$$
$$\sin x = \cos x$$
$$\tan x = 1$$

These are repeats !!

Now, plug in CN's to $f(x) = |\sin x - \cos x|$

$$f(\pi/4) = |\sin \pi/4 - \cos \pi/4| = \left|\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right| = 0$$

$$f(3\pi/4) = \left|\sin\frac{3\pi}{4} - \cos\frac{3\pi}{4}\right| = \left|\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right| = \left|\frac{2\sqrt{2}}{2}\right| = \sqrt{2}. \quad \text{MAX}$$

therefore $0 \leq f(x) \leq \sqrt{2}$

3.7.1 $\begin{cases} \text{Difference} = 128 \\ \text{Prod} = \min \end{cases}$ $\quad x, y$ be the numbers

Constraint: $x - y = 128 \rightarrow \boxed{y = x - 128}$

T.B.O. : $p(x) = xy = x(x-128)$

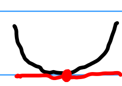
$$p(x) = x^2 - 128x$$

CNs: $p'(x) = 2x - 128 = 0 \qquad x = \frac{128}{2} = 64$

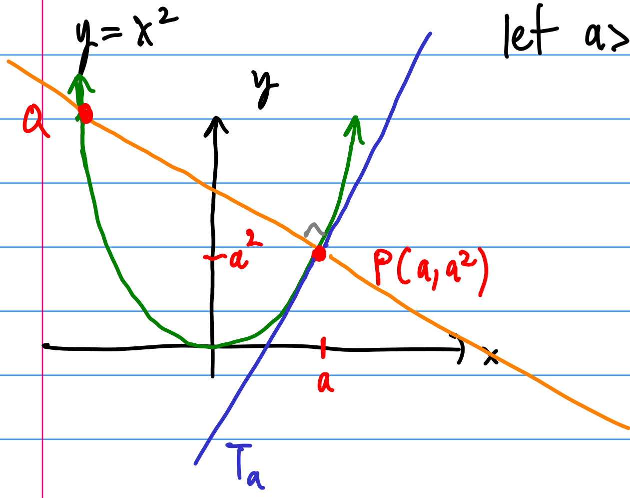
$$y = 64 - 128 = -64$$

$$(x, y) = (64, -64)$$

SDT: $p''(x) = (p'(x))' = 2 > 0$



So the CP is a min.



let $a>0$

$Q(x, y)$ Find the smallest y-value of Q.

$$N_a: \; y = a^2 + \left(\underbrace{\frac{-1}{f'(a)}}\right)(x-a)$$

$y = x^2$

$f'(x) = 2x$ so $f'(a) = 2a$

$$N_a: \; y = a^2 - \frac{1}{2a}(x-a)$$

$$\underline{Q} \quad \begin{cases} y = \frac{-1}{2a}x + a^2 + \frac{1}{2} \\ y = x^2 \end{cases}$$

$$x^2 = \frac{-1}{2a}x + a^2 + \frac{1}{2}$$ Solve for x

$$x^2 + \frac{1}{2a}x + \frac{1}{16a^2} = a^2 + \frac{1}{2} + \frac{1}{16a^2}$$

$$\left(x + \frac{1}{4a}\right)^2 = \left(a + \frac{1}{4a}\right)^2$$

$$x = \frac{-1}{4a} \pm \left(a + \frac{1}{4a}\right)$$

$$= \underbrace{a}_{P}, \; \underbrace{-a - \frac{1}{2a}}_{Q}$$

$$y = \left(-a - \frac{1}{2a}\right)^2 = \left(a + \frac{1}{2a}\right)^2 = y \quad \text{T.B.O.}$$

Now optimize, $$\frac{dy}{da} = \cancel{2}\left(a + \frac{1}{2a}\right)\left(1 - \frac{1}{2a^2}\right) = 0$$

$$a - \cancel{\frac{1}{2a}} + \cancel{\frac{1}{2a}} - \frac{1}{4a^3} = 0$$

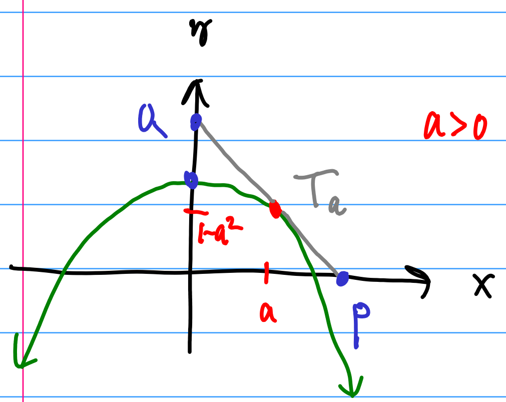
$$a - \frac{1}{4a^3} = 0$$

$$a = \frac{1}{4a^3}$$

$$a^4 = \frac{1}{4} \Rightarrow a^2 = \frac{1}{2} \Rightarrow a = \pm\sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{a = \frac{1}{\sqrt{2}}}$$

Ex. $y = 1 - x^2$ $\qquad y' = -2x$



Q: Find a to minimize the area of the triangle cut off by the tan. line.

$$T_a: \quad y = f(a) + f'(a)(x-a)$$

$$y = 1 - a^2 - 2a(x-a)$$

$$y = -2ax + \underbrace{1 - a^2 + 2a^2}_{y\text{-int}}$$

$Q(0, 1+a^2)$

x-int: $y = 0 = -2ax + 1 + a^2$ solve for x.

$$x = \frac{1+a^2}{2a} \qquad \text{so} \quad P\left(\frac{1+a^2}{2a}, 0\right)$$

TBO. $A(a) = \frac{1}{2}bh = \frac{1}{2}(1+a^2)\left(\frac{1+a^2}{2a}\right) = \frac{1}{4}\frac{(1+a^2)^2}{a}$

$\frac{dA}{da}$ $\qquad$ Do IT.