3.3.8

Ex. $f(x) = 3+12x^2-8x^3$ find local extreme values

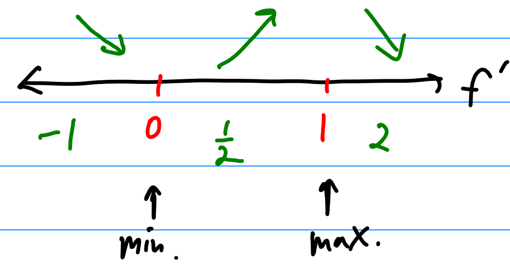
☺ (*) (*) **Fermat's Theorem** – If f has a local max/min at $x=a$, then a is critical number of f.

$f'(x) = 24x - 24x^2 = 0$ solve for x.

$24x(1-x)=0$

$x=0$ $x=1$

F.D.T.



$f'(x) = \cancel{24}\, x\,(1-x)$

$f'(-1) = - + = -$

$f'(\tfrac{1}{2}) = + + = +$

$f'(2) = + - = -$

$f(0) = 3$ ← local min. val.

$f(1) = 7$ ← local max. val.

OR **Second Derivative Test:**

$f''(x) > 0$ local min.

$f''(x) < 0$ local max.

$f''(x) = 0$ Inconclusive.

$f'(x) = 24x - 24x^2$

$f''(x) = 24 - 48x = 24(1-2x)$

$f''(0) = +$ local min.

$f''(1) = -$ local max.

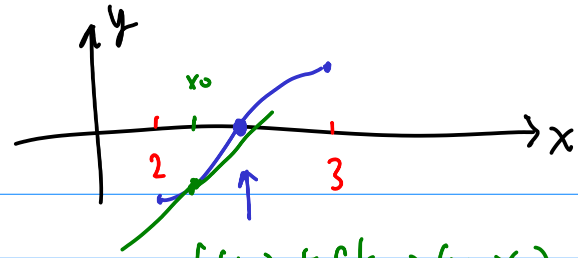
3.8.8 $\underbrace{3x^4-8x^3+6}_{f(x)} = 0$ $[2,3]$

IVT: $f(2) = 3\cdot 2^4 - 8\cdot 2^3 + 6 = 48 - 64 + 6 < 0$

$f(3) = 3\cdot 3^4 - 8\cdot 3^3 + 6 = 243 - 216 + 6 > 0$

Since f is cont on $[2,3]$, and the signs at the endpoints differ then there is a root...

Newton's Method:



$y = f(x_0) + f'(x_0)(x - x_0)$ set $= 0$, solve for x.

for x_k,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$f(x) = 3x^4 - 8x^3 + 6$

$f'(x) = 12x^3 - 24x^2 = 12x^2(x-2)$ can't start w/ 2.

$$\longrightarrow \underbrace{x_{k+1} = x_k - \frac{3x_k^4 - 8x_k^3 + 6}{12x_k^3 - 24x_k^2}}$$ in "Excel"

3.9.22. $s(t)$ = position

$v(t) = s'(t)$ velocity

$a(t) = v'(t) = s''(t)$ acceleration

$s(0) = 0$ ~~mi~~ ft

$v(0) = \cancel{50}$ ~~mph~~ $\frac{220}{3}$

$a(t) = -40$ ~~ft/s^2~~

$$50 \frac{\cancel{mi}}{\cancel{hr}} \cdot \frac{5280 \text{ ft}}{1 \cancel{mi}} \cdot \frac{1 \cancel{hr}}{3600 \text{ sec}} = \frac{220}{3} \frac{ft}{s}$$

$v(t) = -40t + C$ $\quad v(0) = C = \frac{220}{3}$

$v(t) = -40t + \frac{220}{3} \rightarrow 0 = -40t + \frac{220}{3}$ to find stopping time.

$t = \frac{220}{120} = \boxed{\frac{11}{6} \text{ sec}}$

$s(t) = V(t) = \frac{-40}{2}t^2 + \frac{220}{3}t + C$ $\quad \underbrace{s(0) = 0 = C}$

$s(t) = -20t^2 + \frac{220}{3}t$ $\quad s\left(\frac{11}{6}\right) = -20\left(\frac{121}{36}\right) + \frac{2420}{18} = \frac{4840 - 2420}{36}$

$$s\left(\frac{11}{6}\right) = \frac{2420}{36}\text{ ft} \approx 67.2\text{ ft.}$$

PP. $x^2+xy+y^2=12$ Find the max. height of the curve. (y-value)

$$\frac{dy}{dx} = \begin{cases} 0 \\ \text{undef.} \end{cases}$$

Implicit Diff: $\frac{d}{dx}\left[x^2+xy+y^2=12\right]$

$$2x + \left(1y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x+2y) = -2x-y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

set $\frac{dy}{dx} = \begin{cases} 0 & \text{TOP} \\ \text{undef.} & \text{BOT.} \end{cases}$

Top: $2x+y=0$ $\begin{cases} y=-2x \\ x^2+xy+y^2=12 \end{cases}$

FINISH IT.