M242 24 Oct '18

§4.5 Integration by Substitution

Ex. $\int_{-2}^{2} 2x\sqrt{4+x^2}\,dx$ Need: $F(x)$ s.t. $F'(x) = 2x\sqrt{4+x^2}$

$$\boxed{\text{Idea: } F'(u) = \sqrt{u} = u^{1/2} \qquad F(u) = \frac{2}{3}u^{3/2}}$$

let $u = 4+x^2$

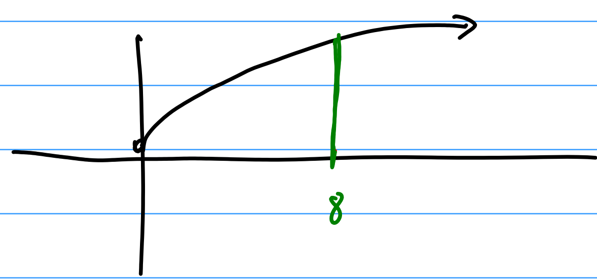
$\boxed{\frac{du}{dx} = 2x}$

$du = 2x\,dx$

$u(2) = 4+2^2 = 8$

$u(-2) = 4+(-2)^2 = 8$

$$= \boxed{\int_8^8 \sqrt{u}\,du} = 0$$



$$\int_8^8 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_8^8 = \frac{2}{3}8^{3/2} - \frac{2}{3}8^{3/2} = 0$$

Ex. $\frac{1}{4}\int 4x^3\cos(x^4+2)\,dx = \frac{1}{4}\int \cos(u)\,du = \frac{1}{4}\sin(u) + C$

$= \frac{1}{4}\sin(x^4+2) + C$

$u = x^4+2$

$du = 4x^3\,dx \longrightarrow \frac{1}{4}du = x^3\,dx$

Ex. $\frac{1}{2}\int \sqrt{2x+1}\, \underbrace{2dx}_{du}$

$u = 2x+1$

$du = 2\,dx \longrightarrow \frac{1}{2}du = dx$

OR

$\int \sqrt{2x+1}\, dx = \int \sqrt{u}\, \frac{1}{2}du$

$= \frac{1}{2}\int \sqrt{u}\, du = \int f(x)\, dx$

f(x)

dx

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{1}{3}(2x+1)^{3/2} + C$

Ex. $\frac{-1}{8}\int \frac{-8x}{\sqrt{1-4x^2}}\, dx = -\frac{1}{8}\int \frac{1}{\sqrt{u}}\, du = -\frac{1}{8}\int u^{-1/2}\, du$

$= -\frac{1}{8}\cdot 2u^{1/2} + C$

$u = 1-4x^2$

$du = -8x\, dx$

$= -\frac{1}{4}\sqrt{1-4x^2} + C$

Ex. $\frac{1}{2}\int \sqrt{1+x^2}\, 2x^5\, dx = \frac{1}{2}\int \sqrt{u}\, x^4\, du$

$\cancel{u = x^5}$

$\cancel{du = 5x^4\, dx}$

$u = 1+x^2 \longrightarrow (x^2)^2 = (u-1)^2$

$du = 2x\, dx$

$= \frac{1}{2}\int \sqrt{u}\,(u-1)^2\, du = \frac{1}{2}\int u^{1/2}(u^2-2u+1)\, du$

$= \frac{1}{2}\int u^{5/2} - 2u^{3/2} + u^{1/2}\, du$

$= \frac{1}{2}\left(\frac{2}{7}u^{7/2} - 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right) + C$

$$= \frac{1}{7}u^{7/2} - \frac{2}{5}u^{5/2} + \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C$$

Ex. $\displaystyle\int_0^{\pi/6} \frac{\sin t}{\cos^2 t}\, dt = \int_0^{\pi/6} \underbrace{\frac{1}{\cos t}} \cdot \underbrace{\frac{\sin t}{\cos t}}\, dt = \int_0^{\pi/6} \underline{\sec t \cdot \tan t}\, dt$

$$= \sec t \Big|_0^{\pi/6} = \sec \pi/6 - \sec 0 = \frac{2}{\sqrt{3}} - 1 = \frac{2-\sqrt{3}}{\sqrt{3}}. \blacksquare$$

$$-\int_0^{\pi/6} -\frac{\sin t}{\cos^2 t}\, dt = -\int_1^{\sqrt{3}/2} \frac{1}{u^2}\, du = \int_{\sqrt{3}/2}^1 \frac{1}{u^2}\, du = \int_{\sqrt{3}/2}^1 u^{-2}\, du$$

$u = \cancel{\sin t}$ $\quad$ $u = \cos t$ $\quad$ $u(\pi/6) = \cos(\pi/6) = \frac{\sqrt{3}}{2}$

$du = \cancel{\cos t\, dt}$ $\quad$ $du = -\sin t\, dt$ $\quad$ $u(0) = \cos(0) = 1$

$$= -u^{-1}\Big|_{\sqrt{3}/2}^{1} = -\frac{1}{1} - \left(-\frac{1}{\sqrt{3}/2}\right) = \frac{2}{\sqrt{3}} - 1 = \frac{2-\sqrt{3}}{\sqrt{3}}.$$