M242 26 Oct 2018

Chapter 4 Review, Part II

1. Integrate:

a.) $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$ c.) $\int_0^{\pi/4} (1+\tan\theta)^3 \sec^2\theta\, d\theta$ e.) $\int x^5 (x^2-1)^4\,dx$

b.) $\int \sin(x)\cos(\cos(x))\,dx$ d.) $\int \sin(\pi t)\cos(\pi t)\,dt$ f.) $\int_1^9 \frac{\sqrt{u}-2u^2}{u}\,du$

2. If $\int_0^x f(t)\,dt = x\sin x + \int_0^x \frac{f(t)}{1+t^2}\,dt$ for all x, find an explicit formula for $f(x)$.

3. $f(x) = \begin{cases} -x-1 & -3 \le x \le 0 \\ -\sqrt{1-x^2} & 0 \le x \le 1 \end{cases}$. Compute $\int_{-3}^1 f(x)\,dx$

Brief Solutions:

1. a.) $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx \left\{ \begin{matrix} u = 1+\sin x \\ du = \cos x\,dx \end{matrix} \right\} = \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = 2u^{1/2}+C = 2\sqrt{1+\sin x}+C$

b.) $-\int -\sin(x)\cos(\cos(x))\,dx \left\{ \begin{matrix} u = \cos x \\ du = -\sin(x)\,dx \end{matrix} \right\} = -\int \cos u\,du = -(-\sin u)+C = \sin(\cos(x))+C$

c.) $\int_0^{\pi/4} (1+\tan\theta)^3 \sec^2\theta\,d\theta \left\{ \begin{matrix} u = 1+\tan\theta & u(\pi/4) = 1+\tan\pi/4 = 1+1 = 2 \\ du = \sec^2\theta\,d\theta & u(0) = 1+\tan 0 = 1+0 = 1 \end{matrix} \right\}$

$= \int_1^2 u^3\,du = \frac{1}{4}u^4\Big|_1^2 = 4 - 1/4 = 15/4$

d.) $\frac{1}{\pi}\int \pi\sin(\pi t)\cos(\pi t)\,dt \left\{ \begin{matrix} u = \sin(\pi t) \\ du = \pi\cos(\pi t)\,dt \end{matrix} \right\} = \frac{1}{\pi}\int u\,du = \frac{1}{2\pi}u^2+C = \frac{1}{2\pi}\sin^2(\pi t)+C$

e.) $\frac{1}{2}\int x^5(x^2-1)^4\,2\,dx \left\{ \begin{matrix} u = x^2-1 \Rightarrow x^2 = u+1 \Rightarrow x^4 = (u+1)^2 \\ du = 2x\,dx \end{matrix} \right\} = \frac{1}{2}\int (u+1)^2 u^4\,du$

$= \frac{1}{2}\int u^6 + 2u^5 + u^4\,du = \frac{1}{2}\left(\frac{1}{7}u^7 + \frac{2}{6}u^6 + \frac{1}{5}u^5\right) + C$

$= \frac{1}{14}(x^2-1)^7 + \frac{1}{6}(x^2-1)^6 + \frac{1}{10}(x^2-1)^5 + C$

f.) $\int_1^9 \frac{\sqrt{u}-2u^2}{u}\,du = \int_1^9 u^{-1/2} - 2u\,du = 2u^{1/2} - u^2\Big|_1^9 = 2(3-1) - (81-1) = 4 - 80 = -76$

2. $\int_0^x f(t)\,dt = x\sin x + \int_0^x \frac{f(t)}{1+t^2}\,dt$

Take d/dx of the equation implicitly:

$$\frac{d}{dx}\int_0^x f(t)\,dt = \frac{d}{dx}\left[x\sin x\right] + \frac{d}{dx}\int_0^x \frac{f(t)}{1+t^2}\,dt$$

$$f(x) = \sin x + x\cos x + \frac{f(x)}{1+x^2}$$

Now solve for $f(x)$:

$$\left(1-\frac{1}{1+x^2}\right)f(x) = \sin x + x\cos x$$

$$\frac{x^2}{1+x^2}f(x) = \sin x + x\cos x$$

$$f(x) = \frac{(1+x^2)(\sin x + x\cos x)}{x^2} = \frac{\sin x + x\cos x + x^2\sin x + x^3\cos x}{x^2}$$

so $\boxed{f(x) = \frac{1}{x^2}\sin x + \frac{1}{x}\cos x + \sin x + x\cos x}$

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3. $\int_{-3}^{1} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_0^1 f(x)\,dx$

$$= \int_{-3}^{0} -x-1\,dx + \boxed{\int_0^1 -\sqrt{1-x^2}\,dx}$$ ← Geometry! 1/4 of a unit circle.

$$= -\frac{1}{2}x^2 - x\Big|_{-3}^{0} + \left(-\frac{1}{4}\pi\right)$$

$$= -\frac{9}{2} + 3 - \frac{1}{4}\pi = \frac{-3}{2} - \frac{\pi}{4} = -\left(\frac{6+\pi}{4}\right)$$