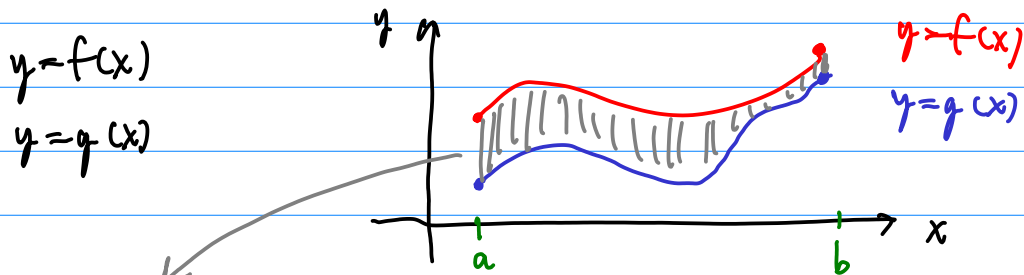


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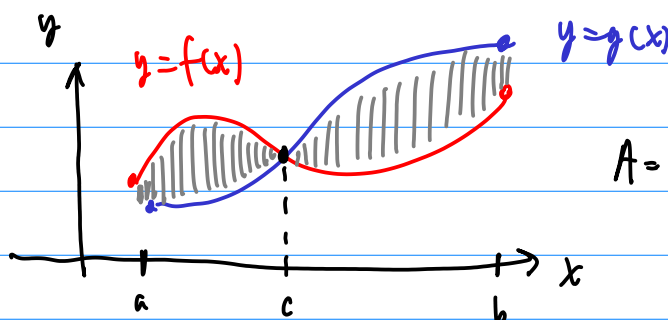
29 Oct '18

§5.1 - Area between curves

Q: What is the area?

$$A = \int_a^b f(x) dx - \int_a^b g(x) dx$$

$$A = \int_a^b [f(x) - g(x)] dx$$

Ex.

$$A = \int_a^c [f(x) - g(x)] dx + \int_c^b [g(x) - f(x)] dx$$

Ex.

$$y = 2x + 2 \quad x = -1, 2$$

$$y = 14 - x^2$$

Set equal:

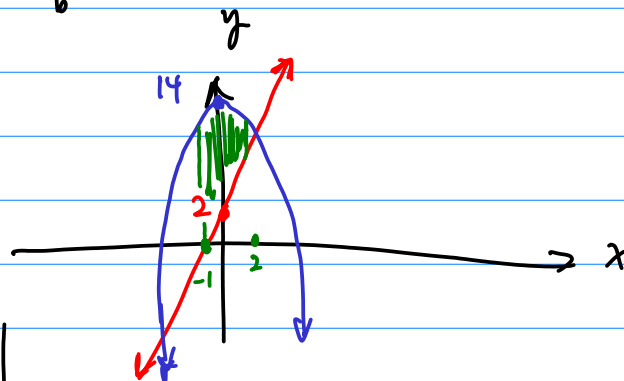
$$2x + 2 = 14 - x^2$$

$$x^2 + 2x - 12 = 0$$

$$x^2 + 2x + 1 = 12 + 1$$

$$(x+1)^2 = 13$$

$$x = -1 \pm \sqrt{13}$$

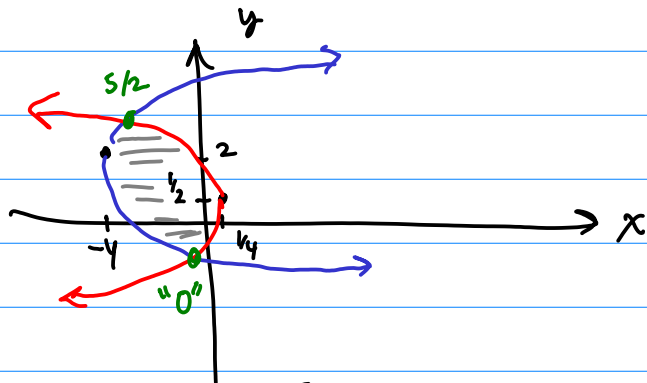
both outside of $[-1, 2]$.

$$\begin{aligned}
 A &= \int_{-1}^2 (14 - x^2 - (2x + 2)) dx \\
 &= \int_{-1}^2 (-x^2 - 2x + 12) dx \\
 &= \left[-\frac{1}{3}x^3 - \frac{2}{2}x^2 + 12x \right]_{-1}^2
 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{8}{3} - 4 + 24 - \left(-\frac{1}{3}(-1) - 1 - 12\right) \\
 &= \left(-\frac{8}{3} - \frac{1}{3}\right) - 4 + 24 + 1 + 12 \\
 &= -3 - 4 + 24 + 1 + 12 = \boxed{30}
 \end{aligned}$$

Ex. $x = y^2 - 4y$ — blue
 $x = y - y^2$ — red

$\int_{?}^{?} [? - ?] dy$



set equal: $y^2 - 4y = y - y^2$
 $2y^2 - 5y = 0$
 $y(2y - 5) = 0$
 $y = 0 \quad y = 5/2$

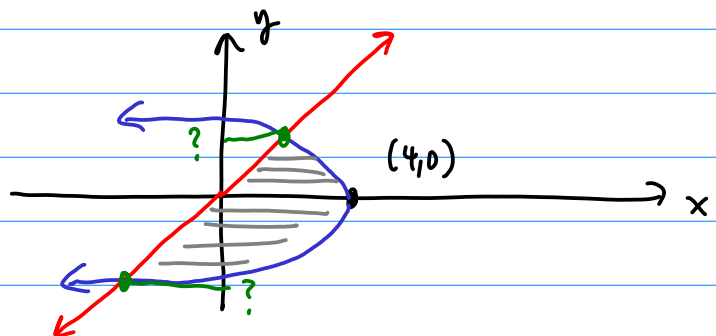
$A = \int_0^{5/2} (y - y^2 - (y^2 - 4y)) dy$
 $= \int_0^{5/2} (-2y^2 + 5y) dy$

$= \left. -\frac{2}{3}y^3 + \frac{5}{2}y^2 \right|_0^{5/2}$
 $= \frac{1}{3} \cdot \left(\frac{5}{2}\right)^3 = \frac{125}{24}$

$-\frac{2}{3}\left(\frac{5}{2}\right)^3 + \frac{5}{2}\left(\frac{5}{2}\right)^2 + 0 - 0$

$= -\frac{2}{3}\left(\frac{5}{2}\right)^3 + 1\left(\frac{5}{2}\right)^3 = \left(1 - \frac{2}{3}\right)\left(\frac{5}{2}\right)^3$
 $= \frac{1}{3} \cdot \left(\frac{5}{2}\right)^3$

Ex. $\begin{cases} 2x + y^2 = 8 \\ x = y \end{cases}$ — blue
— red



$2x + y^2 = 8$
 $2x = -y^2 + 8$
 $x = -\frac{1}{2}y^2 + 4$

$-\frac{1}{2}y^2 + 4 = y$
 $y^2 - 8 = -2y$

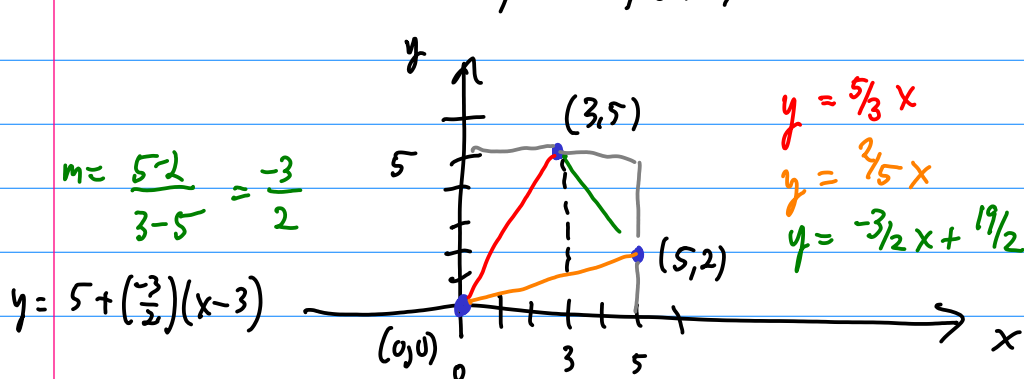
$$y^2 + 2y - 8 = 0$$

$$(y+4)(y-2) = 0$$

$$y = -4, y = 2$$

$$A = \int_{-4}^2 -\frac{1}{2}y^2 + 4 - y \, dy = \text{FTIS} \dots$$

Ex. Use integrals to compute the area of the triangle w/ vertices $(0,0)$, $(5,2)$, $(3,5)$



Geometry

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}ab\sin C$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}p$$

$A = \text{bootstrapping} \leftarrow$

$$A = \int_0^3 \left(\frac{5}{3}x - \frac{2}{5}x\right) dx + \int_3^5 \left(-\frac{3}{2}x + \frac{19}{2} - \frac{2}{5}x\right) dx = \text{FTIS}$$

$$A = \frac{1}{2} \left[(0 \cdot 2 - 5 \cdot 0) + (5 \cdot 5 - 2 \cdot 3) + (5 \cdot 0 - 0 \cdot 3) \right] = \frac{25 - 6}{2} = \frac{19}{2}$$

$$A = \int_0^3 \frac{25-6}{15} x \, dx + \int_3^5 \left(-\frac{(5+4)}{10} x + \frac{19}{2} \right) dx$$

$$= \frac{19}{15} \cdot \frac{1}{2} (3^2 - 0^2) + \left(-\frac{19}{10} \cdot \frac{1}{2} (5^2 - 3^2) + \frac{19}{2} (5 - 3) \right)$$

$$\frac{19 \cdot 9}{15 \cdot 2} - \frac{19(25-9)}{2 \cdot 10} + 19$$

$$(3-8) \cdot \frac{19}{10} + 19 = \frac{19}{2}$$

