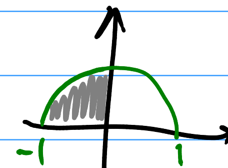
Ch 4. $\int_{-1}^{0} 2+\sqrt{1-x^2}\,dx = \underbrace{\int_{-1}^{0} 2\,dx} + \int_{-1}^{0} \sqrt{1-x^2}\,dx$

~~$u=1-x^2$~~
~~$du=-2x\,dx$~~

$= 2x\Big|_{-1}^{0} + \int_{-1}^{0} \overbrace{\sqrt{1-x^2}}^{y}\,dx$

$y^2=\left(\sqrt{1-x^2}\right)^2$
$y^2=1-x^2$
$x^2+y^2=1^2$



$A=\frac{1}{4}\pi r^2 = \pi/4$

$= 2(0)-2(-1) + \pi/4$
$= 2+\pi/4$

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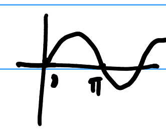
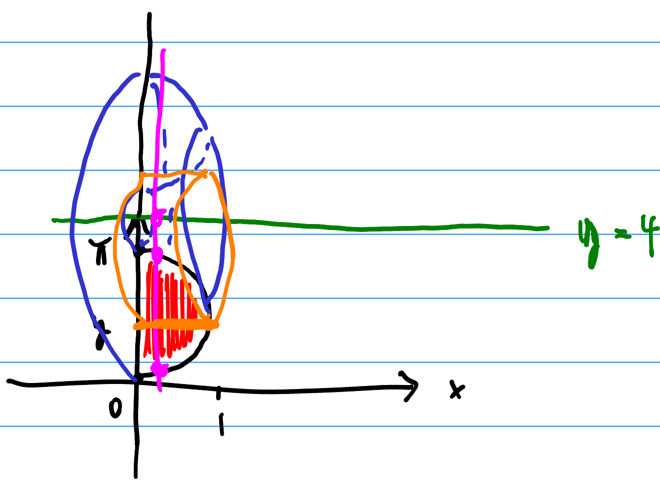
5. Rotate the region bounded between $x=\sqrt{\sin y}$, $0\le y\le\pi$, and $x=0$ about the line $y=4$.

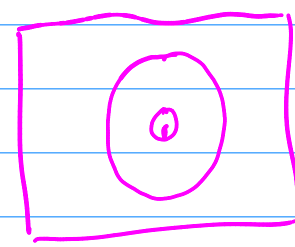
Region: $x=\sqrt{\sin y}$, $0\le y\le \pi$

$x^2=\sin y \to y=\arcsin(x^2)$
$y=\sin^{-1}(x^2)$





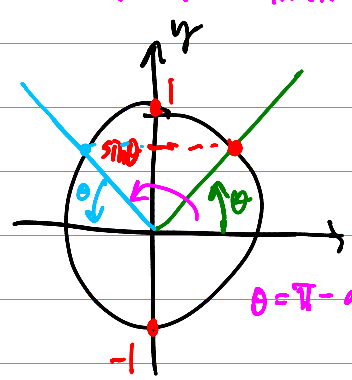
Slice:



$r=4-y=4-(\pi-\arcsin(x^2))$
$R=4-\arcsin(x^2)$

$\to A(x)=\pi R^2-\pi r^2$
$=\pi\left((4-\arcsin(x^2))^2-(4-\pi+\arcsin(x^2))^2\right)$

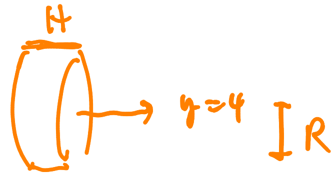
We don't know $\int \arcsin\theta\,d\theta$



$\theta=\pi-\arcsin(\theta)$

Range $(\arcsin(x))=[-\pi/2,\pi/2]$

Shells:



$H = x = \sqrt{\sin y}$

$R = 4 - y$

$A(y) = 2\pi R H = 2\pi (4-y)\sqrt{\sin y}$

$V = 2\pi \int_0^{\pi} (4-y)\sqrt{\sin y}\, dy$ = can't do this either ☹

$u = \sin y$

$du = \cancel{\cos y}\, dy$

(*) - We could approximate by

$2\pi \sum_{i=1}^{n} f(y_i^*)\,\Delta y$ for acceptable n.

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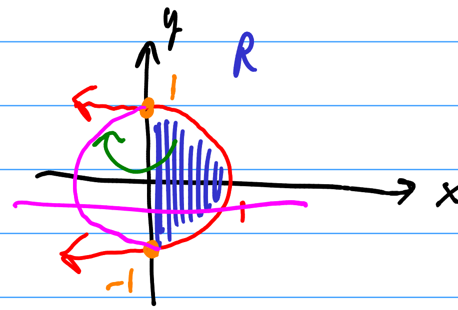
7. $\pi \int_{-1}^{1} \underbrace{(1-y^2)^2}_{R^2}\, dy$ Describe the region for which this integral is the volume.

Looks like slicing!



$R^2 = (1-y^2)^2$

$R = 1 - y^2 = f(y) = x$

$1 - y^2 = 0 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

Slice: $V = \pi \int_a^b R(u)^2\, du$

8. $2\pi \int_1^4 \frac{y+2}{y^2}\, dy$

Shells: $V = 2\pi \int_a^b R(u) H(u)\, du$ ←

Shells,

$\underline{2\pi} \int_1^4 \underbrace{(y+2)}_{R}\ \underbrace{\frac{1}{y^2}}_{H}\, dy$

↑