M242 6 Nov '18

§5.4 - Work

$W = F \cdot d$ work

Newton's 2nd Law: $F = ma = m v' = m s''$

let $F = F(x)$ depend on the position of the particle.

Ex. A book has mass 1.2 kg. Lift the book onto a table 0.7 m off the ground. How much work must be done?

$F = ma = mg \quad g = 9.8\ m/s^2$

$F = (1.2\ kg)(9.8\ m/s^2)$

$W = F \cdot d = (1.2)(0.7)(9.8)\ \frac{kg\, m^2}{s^2} = \frac{kg\, m}{s^2} \cdot m = Nm = J$

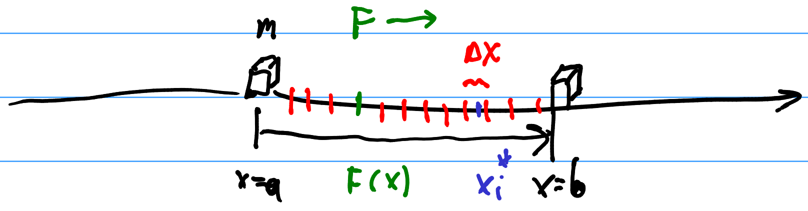
$= \boxed{8.2\ J}$

Ex. 20 lbs ← $F = mg = 20$ lbs

6 ft = d $W = F \cdot d = 20\text{lbs} \cdot 6\text{ft}$

$\boxed{g = 32.1\ ft/s^2}$ ← not needed $= \boxed{120\ \text{ft-lbs}}$

Q. What do we do for changing force? $F = F(x)$



$$W \approx \sum_{i=1}^{n} F(x_i^*)\, \Delta x$$

The exact work done is $W = \lim_{n\to\infty} \sum_{i=1}^{n} F(x_i^*)\,\Delta x = \int_a^b F(x)\,dx.$

Ex. $F(x) = x^2 + 2x$

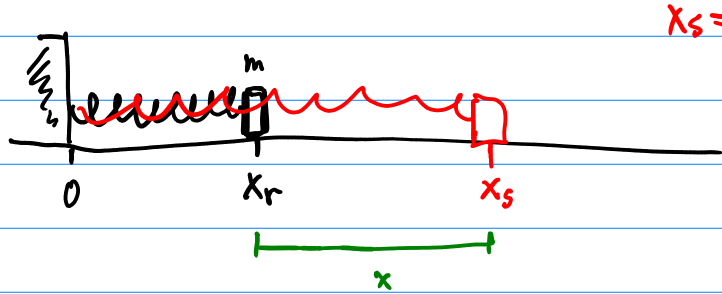
How much work is done for $x: 1 \to 3$?

$$W = \int_1^3 x^2 + 2x \, dx = \frac{1}{3}x^3 + x^2 \Big|_1^3 = 18 - \frac{4}{3} = \frac{50}{3}$$

Ex. Hooke's Law

x_r = resting position

x_s = stretched



$F(x) = kx$

constant

Ex. $F = 40N$ $x = \underbrace{10cm}_{\text{rest}} \to \underbrace{15cm}_{\text{stretched}}$ $x = 15 - 10 = 5$

How much work is done to move the spring from 15cm to 18cm?

Hooke: $F = kx$

$40 = k \cdot 5$

$k = 40/5 = 8$

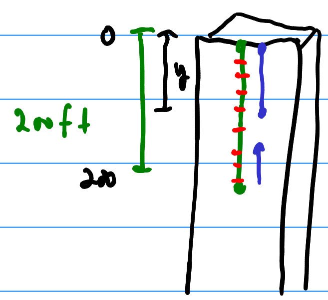
$F(x) = 8x$

$$W = \int_{0.05}^{0.08} 8x\,dx = 4x^2\Big|_5^8 = 4(64-25) = 156 \text{ units}$$

$156 \frac{kg\,cm^2}{s^2}$

So, $\boxed{W = 0.0156\ J}$

Ex. 200 ft long cable weighs 100 lbs w/ constant density.



How much work is done to pull the cable to the top of the building?

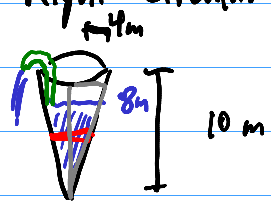
The density is $\frac{100 \text{ lbs}}{200 \text{ ft}} = \frac{1}{2}$ lbs/ft

The weight at any time will be $\frac{1}{2}\,\text{lbs}/\cancel{\text{ft}} \cdot y\,\cancel{\text{ft}} = \frac{1}{2}y$ lbs

The work is thus, $W = \int_0^{200} \frac{1}{2}y\,dy = \frac{1}{4}y^2\Big|_0^{200} = \frac{1}{4}(40000)$

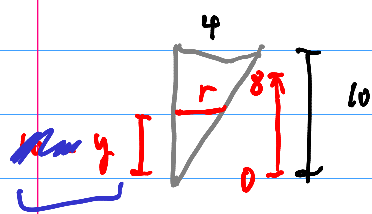
$= 10{,}000$ ft-lbs.

Ex. Right-Circular-Cone Tank



How much work must be done to pump the water?

ρ = density = $1000\ \text{kg}/\text{m}^3$ of H_2O



$$\frac{r}{10-y} = \frac{4}{10} \rightarrow r = \frac{2}{5}(\cancel{10-}y)$$

$$\Delta V = A \approx \pi r^2 = \pi\left(\frac{2}{5}(\cancel{10-}y)\right)^2 = \frac{4\pi}{25}(\cancel{10-}y)^2$$

$$m = \rho \cdot \Delta V = 1000\ \text{kg}/\cancel{\text{m}^3} \cdot \frac{4\pi}{25}(\cancel{10-}y)^2\ \cancel{\text{m}^3}$$

$$m(y) = 160\pi(\cancel{10-}y)^2$$

$$F = m \cdot a = m \cdot g = 160\pi(\cancel{10-}y)^2 \cdot 9.8$$

$$W = \underline{160(9.8)}\,\pi\int_0^8 (\cancel{10-}y)^2\,dy$$

$$W = 1568\pi\int_0^8 (\cancel{10-}y)^2\,dy$$

$$= 1568\pi \cdot \frac{1}{3}8^3 \approx 267{,}605.3\pi\ \text{J}$$