M242 7 Nov '18

5.4.31 Kinetic Energy of an object w/ velocity v and mass m is

$$KE = \frac{1}{2} m v^2$$

$f = f(x)$ force function $\quad f \quad df = f'(t)\,dt$

$x = s(t)$ position function of a particle $\quad x_1 = s(t_1)$ and $x_2 = s(t_2)$

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

Newton's second law:

$$f(x) = m \cdot a(t) = \underbrace{m \frac{dv}{dt}}$$

$$W = \int_{x_1}^{x_2} f(x)\,dx$$

$x = s(t)$

$d(s(t)) = s'(t)\,dt = v(t)\,dt$

$$= \int_{t_1}^{t_2} f(s(t))\,d(s(t)) = \int_{t_1}^{t_2} f(s(t))\,v(t)\,dt$$

$$= \int_{t_1}^{t_2} m \frac{dv}{dt} v(t)\,dt$$

$$= m \int_{t_1}^{t_2} v(t) \frac{dv}{\cancel{dt}} \cancel{dt} \quad = m \int_{t_1}^{t_2} v\,dv$$

$$= \underbrace{m \cdot \frac{1}{2} v^2}\Big|_{t_1}^{t_2}$$

$$= KE(t_2) - KE(t_1)$$

This gives the Work-Energy Theorem: $W = \Delta KE$.

b) How much work in ft-lbs is required to hurl a 12 lb bowling ball at 20 mph?

$v_0 = 0$

$v_1 = 20 \frac{mi}{h} = \frac{20 \cancel{mi}}{1 \cancel{h}} \cdot \frac{5280 \text{ ft}}{1 \cancel{mi}} \cdot \frac{1 \cancel{h}}{3600 \text{ s}}$

$W = KE(v_1) - \underbrace{KE(v_0)}_{0} = KE(v_1) = \frac{1}{2} m v^2$

Not mass

$= \frac{1}{2} \boxed{\cancel{12 \text{ lb}}}$

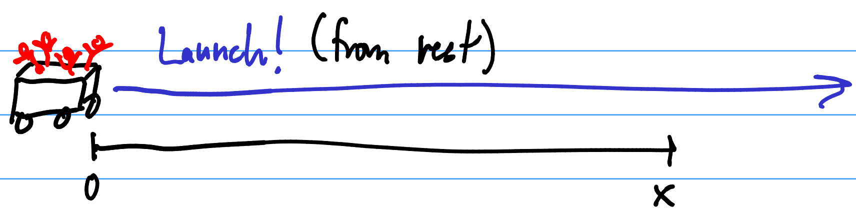
weight

$\frac{12 \text{ lb}}{32 \text{ ft/s}^2} = \frac{3}{8} \frac{\text{lb s}^2}{\text{ft}} = m$

$W = \underbrace{\frac{1}{2} \cdot \frac{3}{\cancel{8}} \cdot \frac{\text{lb} \cancel{s^2}}{\cancel{ft}} \cdot \left(\frac{\cancel{20} \cdot \cancel{5280}^{\cancel{176}\ 88}}{\cancel{3600}_{\cancel{18}\ \cancel{6}\ 3}}\right)^2 \left(\frac{ft}{\cancel{s}}\right)^2}$ ft-lbs

$\frac{11 \cdot \cancel{88}^{44} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{8} \cdot 3} =$

$= \frac{484}{3}$ ft-lbs

5.4.32 

$m = 800 \text{ kg}$

$f(x) = 5.7x^2 + 1.5x \leftarrow$

Use WET to find the velocity when $x = 60$.

$W = \int_0^{60} 5.7x^2 + 1.5x \, dx = \frac{1}{3}(5.7)x^3 + \frac{1.5}{2}x^2 \Big|_0^{60}$

$$= 1.9(60)^3 + 0.75(60^2)$$

$$= 3600(114 + 0.75) = 3600(114.75) = W$$

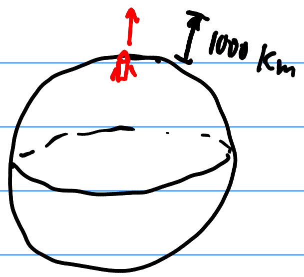
By WET, $W = KE(60) - \cancel{KE(0)}^{\,0}$

$$3600(114.75) = \frac{1}{2} m v^2$$

$$3600(114.75) = \frac{1}{2} 800 \, v^2$$

$$v = \sqrt{9(114.75)} = 3\sqrt{114.75} = 32.14 \text{ m/s}$$

5.4.33



mass of rocket = 1000 kg.

Find the work required to launch.

<u>Newton's Law of Gravity:</u>

m_1 and m_2 masses of two objects

r is the distance between centers of mass

$$F(r) = \frac{m_1 m_2 G}{r^2} \qquad G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Assume object 1 w/ m_1 is fixed, and object 2 is moving.
Find the work done to move O_2 from $r = a$ to $r = b$.

$$W = \int_a^b F(r)\,dr = m_1 m_2 G \int_a^b \frac{1}{r^2}\,dr = m_1 m_2 G \left(\frac{-1}{r}\right)\Bigg|_a^b$$

$$= m_1 m_2 G\left(\frac{-1}{b} + \frac{1}{a}\right) = \boxed{\frac{m_1 m_2 G (b-a)}{ab} = W}$$

$m_E = 5.98 \times 10^{24}$ kg
$r_E = 6.37 \times 10^6$ m
$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

$m_R = 1000$ kg $= 1 \times 10^3$ kg
$r_0 = 1000$ km $= 1{,}000{,}000$ m $= 1 \times 10^6$ m

$a = r_E \qquad b = r_E + r_0$

$a = 6.37 \times 10^6 \qquad b = 7.37 \times 10^6$

$$W = \frac{m_E m_R G (b-a)}{ba} = \frac{(5.98 \times 10^{24})(1 \times 10^3)(6.67 \times 10^{-11})(1 \times 10^6)}{(6.37 \times 10^6)(7.37 \times 10^6)}$$

$$= \frac{(5.98)(6.67)}{(6.37)(7.37)} \times 10^{10} \text{ N}$$

$$= 0.850 \times 10^{10} = \boxed{8.50 \times 10^9 \text{ N}}$$
