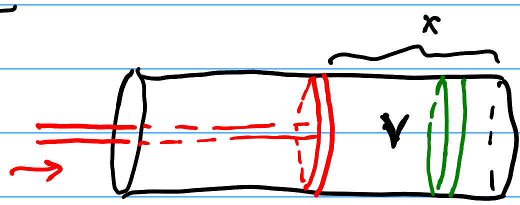
M242 $\qquad$ 8 Nov 18

5.4.29



$P = P(V)$ pressure

$F = \pi r^2 P$ force exerted by gas on piston

$dV = \pi r^2 h = \pi r^2 dx$

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_{x_1}^{x_2} \pi r^2 P(x)\,dx = \boxed{\int_{V_1}^{V_2} P(V)\,dV = W}$$

$V(x_1) = V_1$ $\qquad$ $P(V(x))$ $\qquad$ dV

5.4.30. In a steam engine, $PV^{1.4} = k$, k = constant. (adiabatic expansion)

Calculate the work done $\boxed{P_0 = 160\ \text{lb/in}^2 \quad V_0 = 100\ \text{in}^3}$ $\downarrow$ $V_1 = 800\ \text{in}^3$

Find $W = \int_{100}^{800} P(V)\,dV$ $\qquad$ $PV^{1.4} = k$

$$W = \int_{100}^{800} 160(100)^{1.4}\, V^{-1.4}\,dV$$

$k = 160\,(100)^{1.4}$ = in-lbs

$\boxed{P(V) = 160\,(100)^{1.4} V^{-1.4}}$ TBI

$$= \frac{160(100)^{1.4}}{-0.4} V^{-0.4}\Big|_{100}^{800}$$

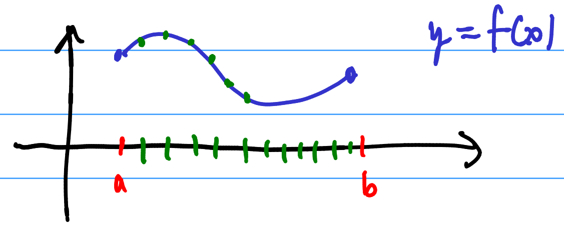
$$= -\frac{160\,(100)^{1.4}}{0.4}\left(800^{-0.4} - 100^{-0.4}\right) = 1882.42 \text{ in-lbs}$$

$$1882.42 \text{ in lbs}\ \frac{1 \text{ ft}}{12 \text{ in}} =$$

$$\boxed{W = 156.87 \text{ ft-lb}} \leftarrow$$

§5.5 – Average Value of a Function

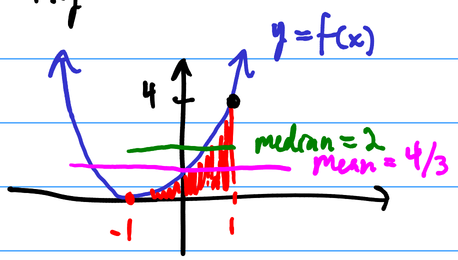
$y = f(x)$ is a continuous function on $[a,b]$



$$f_{avg} \approx \frac{\sum_{i=1}^{n} f(x_i^*)}{n}$$

Actual average value $= \boxed{f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx}$

Ex. Find f_{avg} for $f(x) = x^2+2x+1$ on $[-1,1]$.



$$f_{avg} = \frac{1}{1-(-1)}\int_{-1}^{1} x^2+\cancel{2x}+1\,dx$$

$$= \frac{1}{2}\cdot 2\cdot\int_0^1 x^2+1\,dx$$

$$= \left(\frac{1}{3}x^3+x\right)\Big|_0^1 = \frac{4}{3}$$

The median cuts the height in half.
The mean is the height of the horizontal line that cuts the area under the curve in half.

mean $= f_{avg}$.

Thm. Mean Value Theorem (for Integrals)

Let f be a continuous function on $[a,b]$ and let f_{avg} be its average value on $[a,b]$. Then there exists a number c, $a<c<b$, such that

$$f(c) = f_{avg}. \leftarrow$$

or $$\boxed{f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx} \quad (*)$$

or $$\int_a^b f(x)\,dx = (b-a)\,f(c)$$

Main Idea: We can find the value of c !

Recall: MVT for derivatives :

f is continuous on $[a,b]$

f is differentiable on (a,b)

Then there exists a number c, $a<c<b$, such that

$$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}} \quad (*)$$

→ **Thought Experiment:** Use MVT (D) to prove MVT (I) ! ←

In the meantime, find c for the last example.

$f_{avg} = 4/3$ $\qquad f(x) = x^2+2x+1 \qquad [-1,1]$

$f(x) = f_{avg}$ solve for x

$$x^2+2x+1 = 4/3$$

$$(x+1)^2 = 4/3$$

$$x+1 = \pm\sqrt{4/3}$$

$$x = -1 \pm \frac{2\sqrt{3}}{3}$$

$$\boxed{c = -1 + \frac{2\sqrt{3}}{3}}$$