M242 9 Nov '18

§5.5 Mean Value Theorem for Integrals

MVT(I): f is continuous on $[a,b]$. $a<b$

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Then there exists a number c, $a<c<b$, and

$$f(c) = f_{avg}.$$

Proof. Let F be an antiderivative for f on $[a,b]$.
So $F'(x) = f(x)$ on $[a,b]$.

F is continuous on $[a,b]$ since f is,
F is differentiable on (a,b).

So MVT(D) applies to F: There exists a number c
$a<c<b$, such that

$$f(c) = \boxed{F'(c) = \frac{F(b)-F(a)}{b-a}} = \frac{1}{b-a}\left(F(b)-F(a)\right) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= f_{avg}.$$

so, $f(c) = f_{avg}$. ∎

Practice Exercises

1. Find the average value, f_{avg}, of the function on the interval.
   a.) $f(x) = 3x^2 + 4x$, $[-1, 5]$
   b.) $f(x) = \sqrt{x}$, $[0, 4]$
   c.) $g(t) = \frac{t}{\sqrt{8+t^2}}$, $1 \leq t \leq 8$

2. Find the number(s) c guaranteed by the MVT(I).
   a.) $f(x) = 4\sin x - 2\sin(2x)$ $[0, \pi]$
   (use a calculator to approximate the values of c to 4 decimals)
   b.) $h(x) = 2x - 4$ on $[3, 5]$

3. Find a number b such that $f_{avg} = 4$ on the interval $[-b, b]$ for $f(x) = x^2 + 4x + 8$. Then find the number c that is guaranteed by the MVT(I).