M242 12 Nov '18

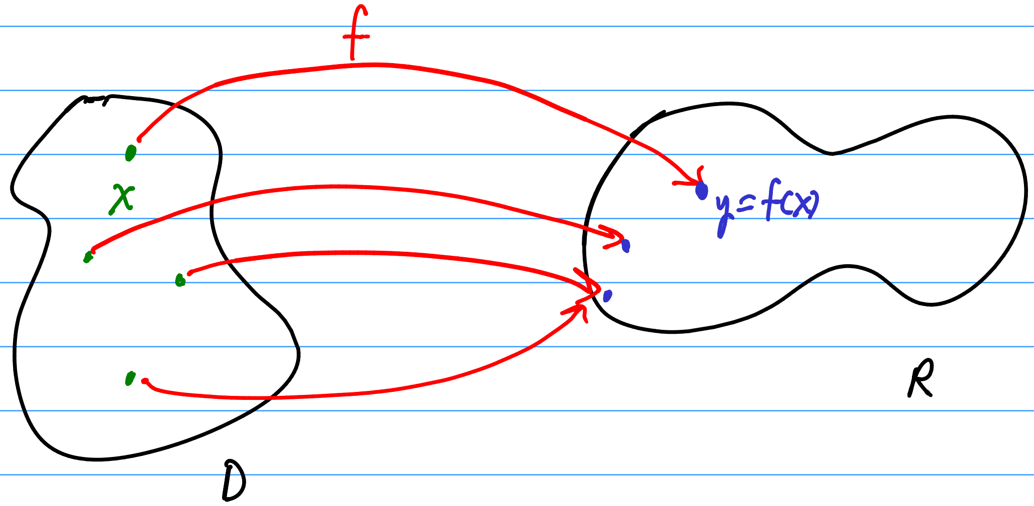
Ch 6 Inverse Functions (and friends)

§6.1 Inverse Function

Def'n. A function is a correspondence between two sets, D and R, such that for each member x in D the corresponds exactly one member y in R.

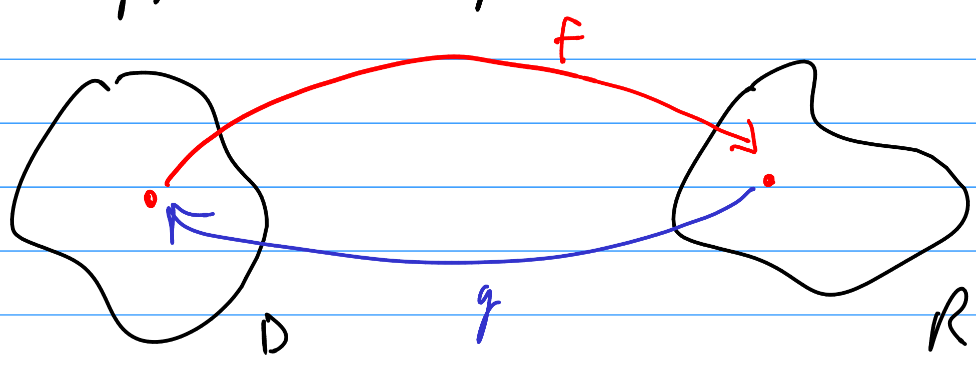
D = domain

R = range



Def'n. Let $f: D \to R$ be a function. The inverse relation g of f satisfies

$$g(y) = x \quad \text{iff} \quad y = f(x)$$



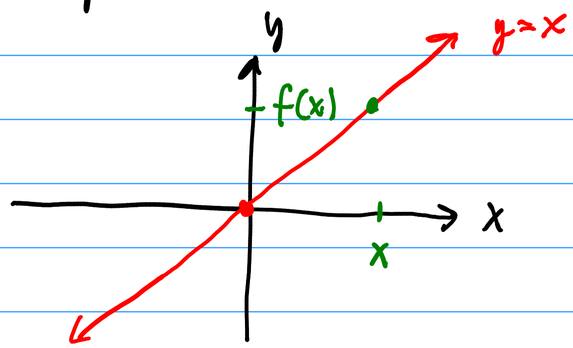
Q. What must be true of f for g to be a function?

Def'n. A function f is said to be <u>one-to-one</u> (1-1) if and only if
$$f(x_1) = f(x_2) \text{ implies that } x_1 = x_2.$$

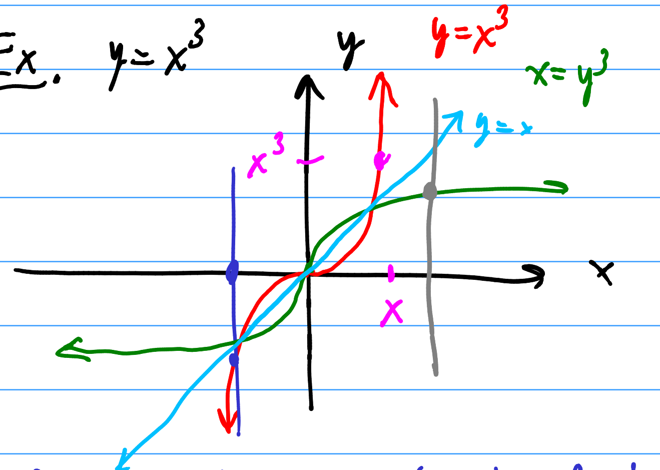
"All the points in the range have only one arrow coming in."

Graphically,

Ex. $y = x$



Ex. $y = x^3$

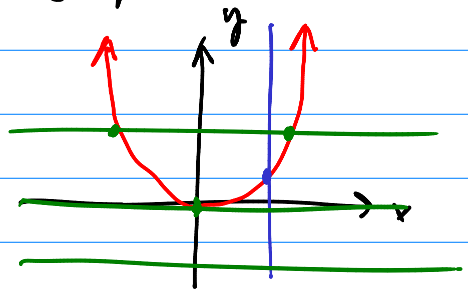


Passes vertical line test (VLT) = function

<u>This</u> inverse relation is a function.

<u>Thm.</u> A function is 1-1 if and only if its graph passes the <u>Horizontal Line Test</u>: For every horizontal line $y=k$, the graph $y=f(x)$ intersects the line <u>at</u> <u>most</u> once.

→ Ex. $y = x^2$



VLT ✓ domain $= \mathbb{R} = (-\infty, \infty)$

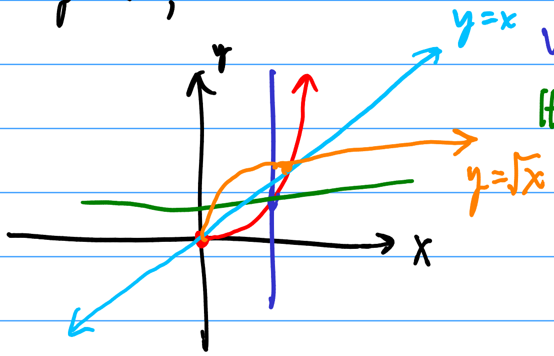
HLT x Not 1-1

Range $= [0, \infty)$

Inverse is not a function.

We can make this function 1-1 by changing the domain:

$y = x^2, \quad x \geq 0$



VLT ✓ $\quad D = [0, \infty)$

HLT ✓ $\quad R = [0, \infty)$

$\Big\}$ 1-1 function

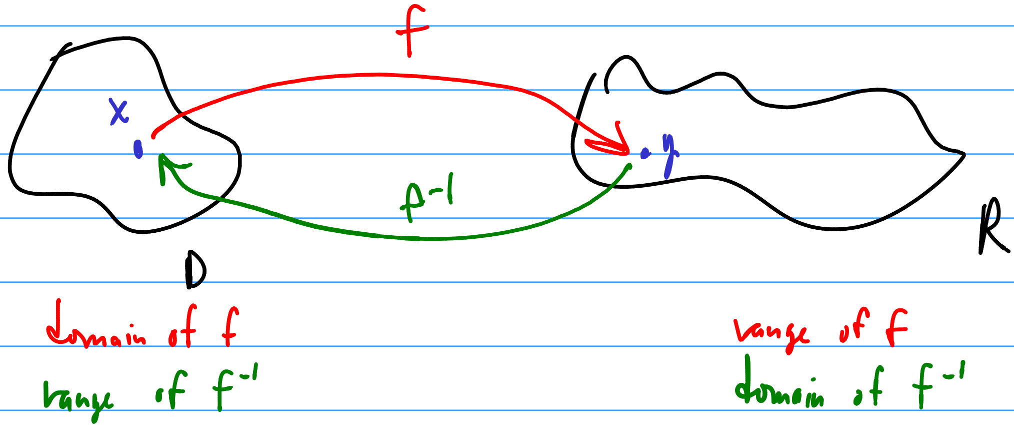
Then $y = x^2, x \geq 0$ has inverse function $y = \sqrt{x}$.

Notation: If f is a 1-1 function, then its inverse function exists, and we write $f^{-1}(x)$ to denote it.

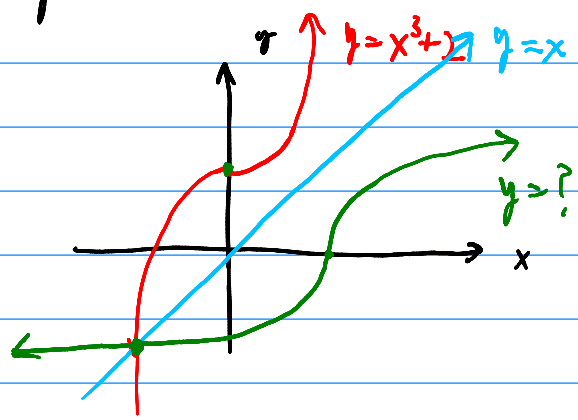
NOTE: $\quad f^{-1}(x) \neq \dfrac{1}{f(x)}$!

Defining Properties: Let f be a one-to-one function w/ domain D and range R. Then

1. $f^{-1}(f(x)) = x \quad$ for all x in D.
2. $f(f^{-1}(x)) = x \quad$ for all x in R.



Ex. $y = x^3+2$ Is it 1-1? If so, find a formula for its inverse.



1-1 by inspection!

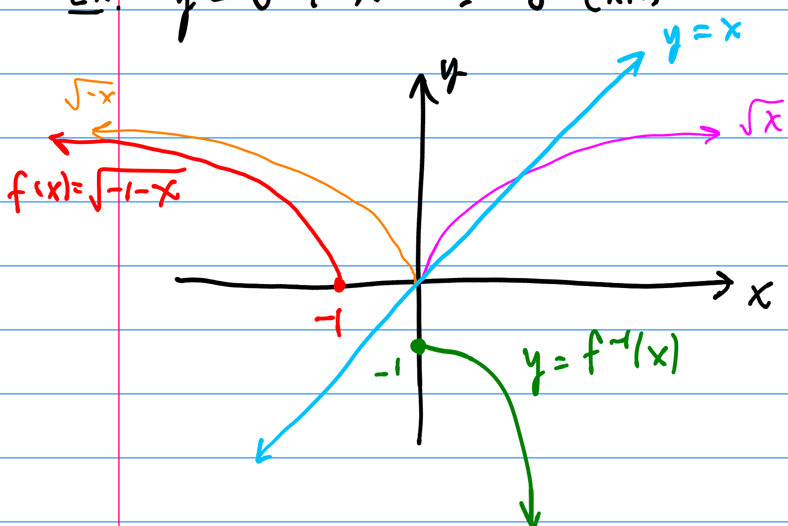
$f(x) = y = x^3+2$

1. swap x and y: $x = y^3+2$
2. Solve for y: $y^3 = x-2$

$y = \sqrt[3]{x-2}$

3. $\boxed{f^{-1}(x) = \sqrt[3]{x-2}}$

Ex. $y = \sqrt{-1-x} = \sqrt{-(x+1)}$



VLT, HLT ✓ 1-1!

$D = (-\infty, -1]$

$R = [0, \infty)$

0. $y = \sqrt{-1-x}$
1. $x = \sqrt{-1-y}$
2. $x^2 = -1-y$

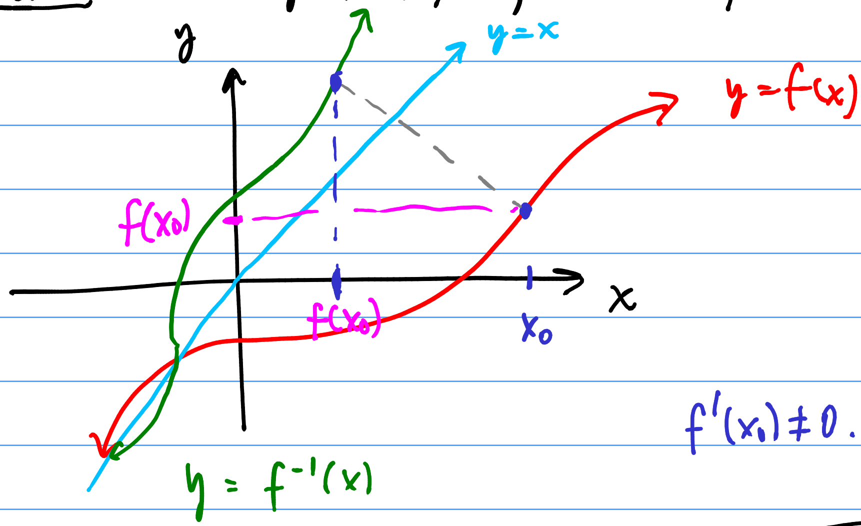
$y = -x^2-1$

3. $f^{-1}(x) = -x^2-1$

But there is a domain restriction.

$f^{-1}(x) = -x^2-1, \quad x \geq 0$, or $[0,\infty)$

Thought Exercise, $y = f(x)$, 1-1, continuous, differentiable.



$f'(x_0) \neq 0.$

$$\boxed{\text{Describe } (f^{-1})'(f(x_0))}$$