M242 16 Nov '18

§6.1–6.3* – Natural Log and Exponential – Exercises.

1. Compute the derivatives of the functions.

a.) e^{-x^2} b.) $\ln(x^2-9)$ c.) $(\ln x)^2-9$

d.) $\ln(\sec x)$ e.) $x^2 e^{\sin x}$ f.) $e^x \cos x$

g.) $e^x \ln x^2$ h.) $e^{2\ln x}$ i.) $\frac{e^x}{\ln x}$

2. Find the antiderivatives of the functions.

a.) $\frac{\ln x}{x}$ b.) $\frac{1}{x\ln x}$ c.) e^{x+e^x}

d.) $x^2 e^{-x^3}$ e.) $\frac{1}{x}\sin(\ln x)$ f.) $e^{2x^2+\ln x}$

3. Find the function $y=y(x)$ that satisfies the initial value problem.

a.) $\begin{cases} dy/dx = 3y \\ y(0)=1 \end{cases}$ b.) $\begin{cases} x\frac{dy}{dx}=1 \\ y(1)=3 \end{cases}$ c.) $\begin{cases} dy/dx = 2y-3 \\ y(0)=0 \end{cases}$

Brief Solutions

1. a.) $(e^{-x^2})' = -2xe^{-x^2}$ b.) $(\ln(x^2-9))' = \frac{2x}{x^2-9}$ c.) $((\ln x)^2-9)' = 2\ln x\cdot\frac{1}{x}$

d.) $(\ln(\sec x))' = \frac{\sec x\tan x}{\sec x} = \tan x$ e.) $(x^2e^{\sin x})' = 2xe^{\sin x} + x^2\cos x\, e^{\sin x}$ f.) $(e^x\cos x)' = e^x\cos x - e^x\sin x$

g.) $(e^x\ln x^2)' = e^x\ln x^2 + \frac{2}{x}e^x$ h.) $(e^{2\ln x})' = (e^{\ln x^2})' = (x^2)' = 2x$ i.) $\left(\frac{e^x}{\ln x}\right)' = \frac{e^x\ln x - \frac{1}{x}e^x}{(\ln x)^2}$

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2. a.) $\int \ln x\,\frac{1}{x}\,dx$ $\left\{\begin{matrix} u=\ln x \\ du = 1/x\,dx\end{matrix}\right\}$ $= \int u\,du = \frac{1}{2}u^2+C = \frac{1}{2}(\ln x)^2+C$

b.) $\int \frac{1}{\ln x}\frac{1}{x}\,dx$ $\left\{\begin{matrix} u=\ln x \\ du = 1/x\,dx\end{matrix}\right\}$ $= \int\frac{1}{u}\,du = \ln(u)+C = \ln(\ln x)+C$

c.) $\int e^{x+e^x}dx = \int e^{e^x}e^x\,dx$ $\left\{\begin{matrix} u=e^x \\ du = e^x\,dx\end{matrix}\right\}$ $= \int e^u\,du = e^u+C = e^{e^x}+C$

d.) $\int x^2e^{-x^3}dx$ $\left\{\begin{matrix} u=x^3 \\ du = 3x^2\,dx\end{matrix}\right\}$ $= \frac{1}{3}\int e^{-u}\,du = -\frac{1}{3}e^{-u}+C = -\frac{1}{3}e^{-x^3}+C$

e.) $\int \sin(\ln x)\,\frac{1}{x}\,dx$ $\left\{\begin{matrix} u=\ln x \\ du = 1/x\,dx\end{matrix}\right\}$ $= \int \sin u\,du = -\cos u + C = -\cos(\ln x)+C$

f.) $\int e^{2x^2+\ln x}\,dx = \int e^{2x^2} e^{\ln x}\,dx = \int e^{2x^2} x\,dx$ $\left\{\begin{array}{l} u = 2x^2 \\ du = 4x\,dx \end{array}\right\}$ $= \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C = \frac{1}{4}e^{2x^2} + C$

3. a.) $dy/dx = 3y \Rightarrow \int \frac{1}{y}\,dy = \int 3\,dx \Rightarrow \ln y = 3x + C$

$y(0) = 1 \Rightarrow \ln(1) = 3(0) + C \Rightarrow 0 = C$

so $\ln y = 3x$ and $\boxed{y = e^{3x}}$.

b.) $x\frac{dy}{dx} = 1 \Rightarrow \int dy = \int \frac{1}{x}\,dx \Rightarrow y = \ln|x| + C$

$y(1) = 3 \Rightarrow \ln|1| + C = 3 \Rightarrow C = 3.$

So, $\boxed{y = \ln|x| + 3}$

c.) $\frac{dy}{dx} = 2y - 3 \Rightarrow \frac{dy}{dx} = 2\left(y - \frac{3}{2}\right) \Rightarrow \int \underbrace{\frac{1}{y - 3/2}}\,dy = \int 2\,dx$

$u = y - 3/2$
$du = dy$

$\Rightarrow \ln\left|y - \frac{3}{2}\right| = 2x + C$

So $y - 3/2 = Ce^{2x}$ and $y = Ce^{2x} + 3/2$.

$y(0) = Ce^0 + 3/2 = C + 3/2 = 0 \Rightarrow C = \frac{-3}{2}.$

and $\boxed{y = \frac{-3}{2}e^{2x} + \frac{3}{2}}$