M242 20 Nov'18

Ex. $y = \log_2 x$ $\quad \frac{dy}{dx} = \frac{1}{x \ln 2}$

Recall. $y = 2^x = (e^{\ln 2})^x$

$= e^{x \ln 2}$

$e^{\ln a} = a$

$(e^u)' = u' e^u$

So $(2^x)' = (e^{x \ln 2})' = \ln 2 \cdot e^{x \ln 2} = \ln 2 \cdot 2^x$

$(2^x)' = \ln 2 \cdot 2^x$

Ex. $y = \log_2 x$ means $x = 2^y$

Implicitly: $\frac{d}{dx}[x = 2^y] \rightarrow 1 = \ln 2 \cdot 2^y \cdot \frac{dy}{dx}$

Solving for $\frac{dy}{dx}$ we get $\frac{dy}{dx} = \frac{d}{dx}[\log_2 x] = \frac{1}{x \ln 2}$

Ex. $\log_2 8 = y \iff 2^y = 8 = 2^3$ $\quad y = 3.$

Ex. $\log 0.1 = \log_{10} 0.1$ $\quad \underbrace{10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01}$

$= \log_{10} \frac{1}{10}$

$= -1$

Ex. $\int 2^x\,dx = \frac{1}{\ln 2}\, 2^x + C$ $(2^x)' = \ln 2 \cdot 2^x$

$$\left(\frac{1}{\ln 2}\cdot 2^x + C\right)' = \frac{1}{\ln 2}\cdot \ln 2\; 2^x = 2^x \quad \checkmark$$

by a u-sub instead:

$$\int 2^x\,dx = \frac{1}{\ln 2}\int e^{x\ln 2}\,\ln 2\,dx = \frac{1}{\ln 2}\int e^u\,du = \frac{1}{\ln 2}e^u + C$$

$u = x\ln 2$

$du = \ln 2\; dx$

$$= \frac{1}{\ln 2}e^{x\ln 2} + C$$

$$= \frac{1}{\ln 2}\,2^x + C$$

Ex. $\int \ln x\,dx = ?$ $\int \frac{1}{x}\,dx = \ln x + C$

Idea: $\int (uv)'\,dx = \int u'v\,dx + \int v'u\,dx$ $u'dx = du$, $v'dx = dv$

$$uv = \int v\,du + \int u\,dv$$

$\longrightarrow$ $\int u\,dv = uv - \int v\,du$ Integration by Parts.

$\int \ln x\,dx$ $u = \ln x$ $\int dv = \int dx$

$du = \frac{1}{x}\,dx$ $v = x$

$= x\ln x - \int x\,\frac{1}{x}\,dx$

$$\int \ln x\,dx = x\ln x - \int 1\,dx = x\ln x - x + C$$

$$\boxed{\int \ln x\,dx = x\ln x - x + C}$$ ⊛

Ex. $\int 3x^2 \ln(x)\, dx = \int \ln x^{3x^2}\, dx = \frac{1}{3}\int 3x^2 \ln(x^3)\, dx$

$\cancel{u = 3x^2}$ $\quad$ $\cancel{u = \ln x}$ $\quad$ $u = x^3$

$\cancel{du = 6x\, dx}$ $\quad$ $\cancel{du = \frac{1}{x}\, dx}$ $\quad$ $du = 3x^2\, dx$

$$= \frac{1}{3}\int \ln u\, du = \frac{1}{3}\left(u \ln u - u\right) + C$$

$$\boxed{= \frac{1}{3}x^3 \ln(x^3) - \frac{1}{3}x^3 + C}$$