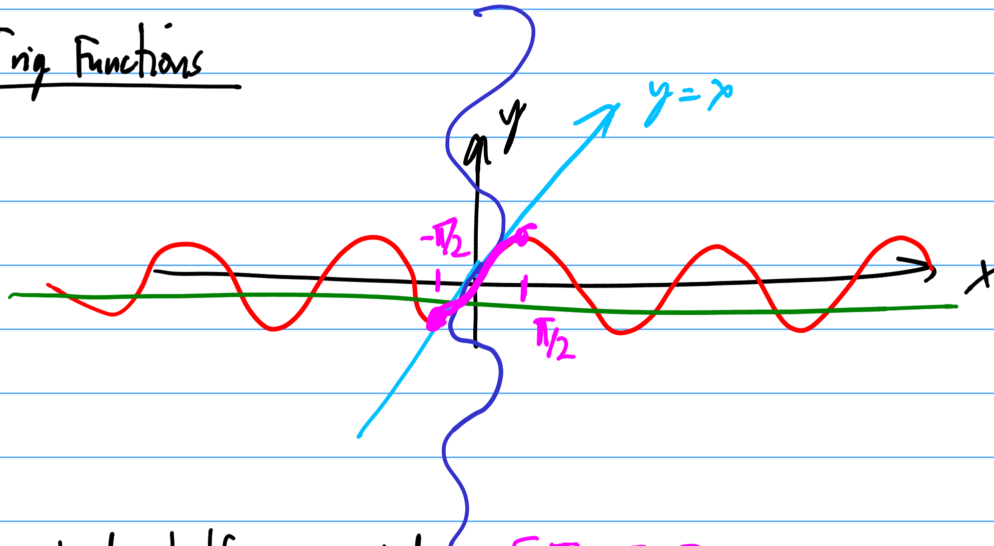
M242 27 Nov '18

## §6.6 Inverse Trig Functions

$y = \sin x$




NOT 1-1

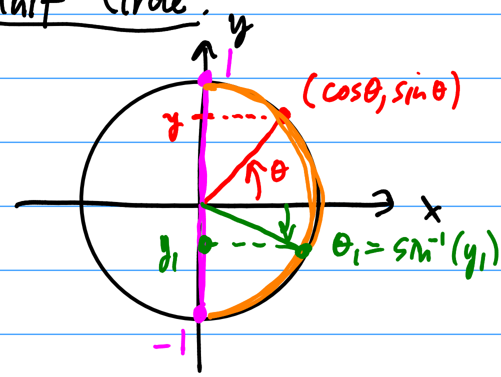
Restrict the graph to half a period: $[-\pi/2, \pi/2]$

Then on this restricted domain we define

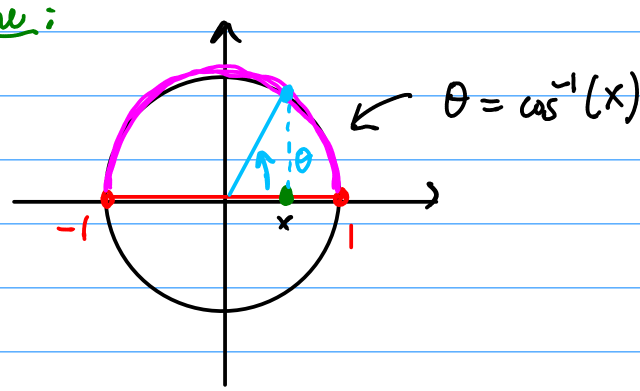
$$y = \sin^{-1}(x) = \arcsin(x) \quad \text{means} \quad \sin(y) = x$$

| | Domain | Range |
|---|---|---|
| $y = \sin(x)$ | $\cancel{\mathbb{R}}$, $[-\pi/2, \pi/2]$ | $[-1,1]$ |
| $y = \sin^{-1}(x)$ | $[-1,1]$ | $[-\pi/2, \pi/2]$ |

Unit Circle:



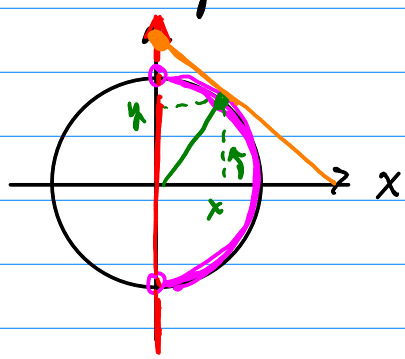

Arccosine:

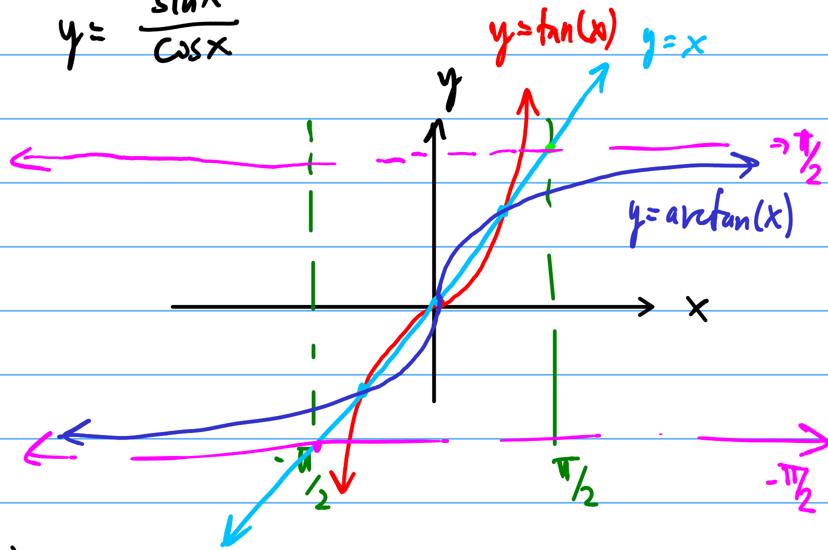



| | domain | Range |
|---|---|---|
| $y = \arccos(x)$ | $[-1,1]$ | $[0,\pi]$ |

## Arctangent:

$y = \tan x \Rightarrow y = \dfrac{\sin x}{\cos x}$




| $y = \arctan(x)$ | domain | range |
|---|---|---|
| | $\mathbb{R}$ | $(-\pi/2, \pi/2)$ |

One period, 1-1!

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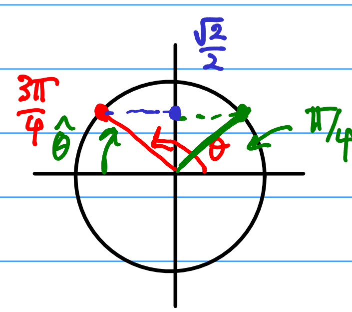
## Inverse Properties:

$\sin(\arcsin(x)) = x$ for $-1 \le x \le 1$

$\arcsin(\sin(x)) = x$ for $-\pi/2 \le x \le \pi/2$

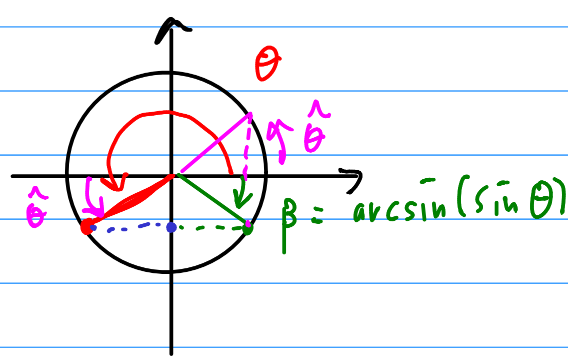
**Ex.** $\arcsin(\sin(\frac{3\pi}{4})) \neq \frac{3\pi}{4}$ ?

$\arcsin(\sin(\frac{3\pi}{4})) = \pi/4$




In QII: $\theta = \pi - \hat{\theta}$

In QIII:




$\theta = \pi + \hat{\theta}$

$\theta = \pi - \beta$

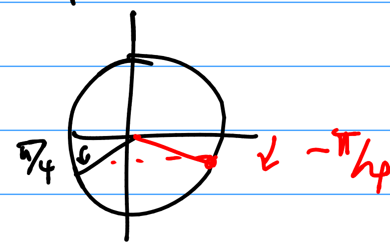
So in QII, QIII, $\arcsin(\sin\theta) = \beta$ and $\theta = \pi - \beta$.

Ex. $\sin^{-1}\left(\sin\left(\frac{13\pi}{4}\right)\right) = \sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right) = -\pi/4$

$\cancel{2}\pi + \frac{5\pi}{4}$

Q3

$\frac{5\pi}{4} = \pi + \frac{\pi}{4}$

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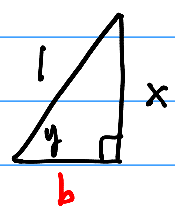
Ex. Find $\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{d}{dx}\left[\arcsin(x)\right]$

$y = \arcsin(x) \Rightarrow \frac{d}{dx}\left[\sin(y) = x\right]$ Implicit derivative.

$\sin(y) = \frac{x}{1}$

$\cos(y) \cdot \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$




$\cos(y) = \frac{A}{H} = \sqrt{1-x^2}$

$1^2 = b^2 + x^2$

$b = \sqrt{1-x^2}$

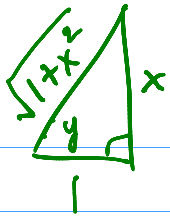
$$\boxed{\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}} \quad (*)$$

Domain : $(-1, 1)$

Ex. $\frac{d}{dx}\left[\tan^{-1}(x)\right]$

$y = \tan^{-1}(x) \Rightarrow \tan(y) = x \Rightarrow \frac{d}{dx}\left[\tan(y) = x\right]$

$\Rightarrow \sec^2(y)\frac{dy}{dx} = 1$




$\tan y = \frac{x}{1}$

$\sec y = \frac{H}{A} = \sqrt{1+x^2}$

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)} = \frac{1}{1+x^2}$$

So, $$\frac{d}{dx}\left[\arctan(x)\right] = \frac{1}{1+x^2}$$

$D: \mathbb{R}$