M242 $\qquad\qquad$ 28 Nov '18

$$y = \arcsin(x) \longrightarrow \frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

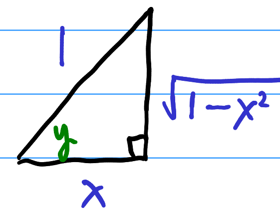
$$y = \tan^{-1}(x) \longrightarrow \frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}$$

$$y = \arccos(x) \longrightarrow \frac{d}{dx}\left[x = \cos(y)\right]$$

$$1 = -\sin(y)\,\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-1}{\sin y}$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}}$$



$$1^2 = x^2 + a^2$$

Ex. $y = \arcsec(x) \longrightarrow \sec(y) = \frac{x}{1} \longrightarrow \cos(y) = \frac{1}{x}$

$y = \arccos\left(\frac{1}{x}\right) \longleftarrow$ $\quad [0, \pi]$

Now, $$\frac{d}{dx}\left[\arcsec(x)\right] = \frac{d}{dx}\left[\arccos\left(\underbrace{\tfrac{1}{x}}_{u}\right)\right]$$

$$(\arccos(u))' = \frac{-1}{\sqrt{1-u^2}} \cdot u'$$

$$= \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \frac{-1}{x^2} \longleftarrow$$

x $\quad x$

$$= \frac{1}{x\sqrt{x^2\left(1-\left(\frac{1}{x}\right)^2\right)}}$$

$$\boxed{\frac{d}{dx}\left[\arcsec(x)\right] = \frac{1}{x\sqrt{x^2-1}}}$$

Ex. Compute $\frac{d}{dx}[\operatorname{arccsc}(x)]$ and $\frac{d}{dx}[\operatorname{arccot}(x)]$

Chain Rule Method:

$$\operatorname{arccsc}(x) = \arcsin\left(\frac{1}{x}\right)$$

$$\frac{d}{dx}[\operatorname{arccsc}(x)] = \frac{d}{dx}\Big[\arcsin\big(\underbrace{\tfrac{1}{x}}_{u}\big)\Big] = \frac{1}{\sqrt{1-(\frac{1}{x})^2}} \cdot \underbrace{\frac{-1}{\boxed{x^2}}}_{u'}$$

$x^2 = |x|\,|x|$

$$\boxed{\frac{d}{dx}[\operatorname{arccsc}(x)] = \frac{-1}{|x|\sqrt{x^2-1}}}$$

Ex. $\frac{d}{dx}[\operatorname{arccot}(x)] = \frac{d}{dx}\left[\arctan\left(\frac{1}{x}\right)\right] = \frac{u'}{1+u^2} = \frac{1}{1+(\frac{1}{x})^2} \cdot \frac{-1}{x^2}$

$$\boxed{\frac{d}{dx}[\operatorname{arccot}(x)] = \frac{-1}{x^2+1}}$$

Ex. $y = \sin(\underbrace{\arctan(\underbrace{x^2}_{N})}_{u})$

1. Chain Rule:

$f = \sin u \qquad u = \arctan(N) \qquad N = x^2$

$f' = \cos(u) \qquad u' = \frac{1}{1+N^2} \qquad N' = 2x$

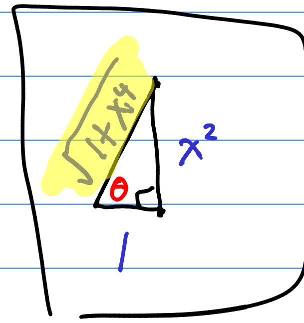
$$\boxed{y' = \underbrace{\cos(\arctan(x^2))}_{\frac{1}{\sqrt{1+x^4}}} \cdot \frac{2x}{1+x^4}}$$

Same! ✓

2. $y = \sin(\underbrace{\arctan(x^2)}_{\theta})$

$\theta = \arctan(x^2)$

$\tan\theta = \frac{x^2}{1}$



$y = \sin\theta = \frac{x^2}{\sqrt{1+x^4}}$

Now compute $\frac{d}{dx}\left[\frac{x^2}{\sqrt{1+x^4}}\right]$ Q Rule!

$(\sqrt{1+x^4})' = \sqrt{u}'$

$= \frac{u'}{2\sqrt{u}}$

$= \frac{\cancel{2}\,\cancel{4}2x^3}{\cancel{2}\sqrt{1+x^4}}$

$$= \frac{\frac{\sqrt{1+x^4}^{\,2}(2x)}{1\cdot\sqrt{1+x^4}} - \frac{x^2\cdot 2x^3}{\sqrt{1+x^4}}}{(\sqrt{1+x^4})^2}$$

$$= \frac{(1+x^4)(2x) - 2x^5}{\sqrt{1+x^4}^{\,3}} = \boxed{\frac{2x}{\sqrt{1+x^4}^{\,3}}}$$

Ex. $-\int \underbrace{\frac{-1}{\sqrt{1-x^2}}} dx = \arcsin(x) + C_1$

$= -\arccos(x) + C_2$

$[-\pi/2, \pi/2] + C$

$[0, \pi] + C$

Ex. $-\int \frac{-\sin x}{1+\cos^2 x} dx$

$\cos^2 x + \sin^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$u = \cos x$

$du = -\sin x \, dx$

$= -\int \frac{1}{1+u^2} du = -\arctan(u) + C$

$$= -\arctan(\cos x) + C$$

Ex. $\displaystyle\int \frac{1}{x\sqrt{1-(\ln x)^2}} \cdot 1 \, dx = \int \frac{1}{\sqrt{1-u^2}}\, du = \arcsin(u) + C$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$= \arcsin(\ln x) + C$$

Ex. $\displaystyle\int \frac{e^{x^2} \cdot 2x}{1 + e^{2x^2}}\, dx$

$e^2 e^{x^2} = e^{2+x^2}$

$e^2 + e^{x^2}$

$(e^{x^2})^2 = e^{2x^2}$

$u = e^{2x^2}$

$du = 4x\, e^{2x^2}\, dx$

$u = e^{x^2}$

$du = 2x\, e^{x^2}\, dx$

$$\int \frac{1}{1+u^2}\, du = \arctan(u) + C$$

$$= \arctan(e^{x^2}) + C$$
