13. Question Details

Find dy/dx by implicit differentiation.

$$\tan^{-1}(5x^2y) = x + 2xy^2$$

$y' =$

$$\frac{d}{dx}\left[\arctan(5x^2y) = x + 2xy^2\right]$$

$$\frac{10xy + 5x^2\, dy/dx}{1+(5x^2y)^2} = 1 + 2y^2 + 2x\cdot 2y \cdot \frac{dy}{dx}$$

$$10xy + \underline{5x^2\frac{dy}{dx}} = \left(1 + 2y^2 + 4xy\frac{dy}{dx}\right)\left(1+25x^4y^2\right)$$

$$= (1+25x^4y^2) + 2y^2(1+25x^4y^2) + \underline{4xy(1+25x^4y^2)\frac{dy}{dx}}$$

$$\left(5x^2 - 4xy - 100x^5y^3\right)\frac{dy}{dx} = 1 + 25x^4y^2 + 2y^2 + 50x^4y^4 - 10xy$$

$$\boxed{\frac{dy}{dx} = \frac{1+25x^4y^2+2y^2+50x^4y^4-10xy}{5x^2-4xy-100x^5y^3}}$$

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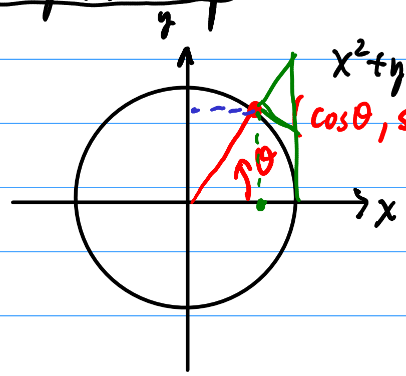
§6.7 & 6.8 – Hyperbolic Trig.

Regular Trig.

$s = r\theta$



$x^2+y^2=1$

$(\cos\theta, \sin\theta)$

$\parallel$

$\underbrace{(\cos(2A), \sin(2A))}$

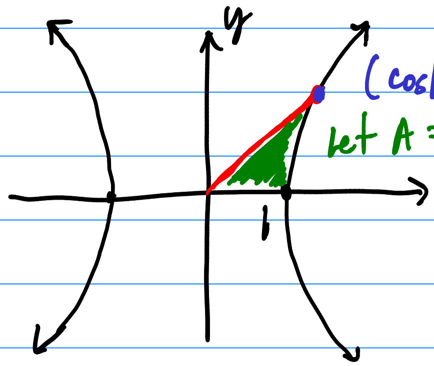
$\theta =$ angle measure

we could swap angle measure for arc length.

Area of a sector: $A = \frac{1}{2}\theta$

$\theta = 2A$

Unit hyperbola: $x^2 - y^2 = 1$



$(\cosh(\tau), \sinh(\tau))$

let A = area and put $\tau = 2A$ or $A = \frac{\tau}{2}$.

$x = \cosh(\tau)$
$y = \sinh(\tau)$

Pythagorean-ish Identity: $\cosh^2\tau - \sinh^2\tau = 1$

Alternate definitions of hyp. trig. functions.

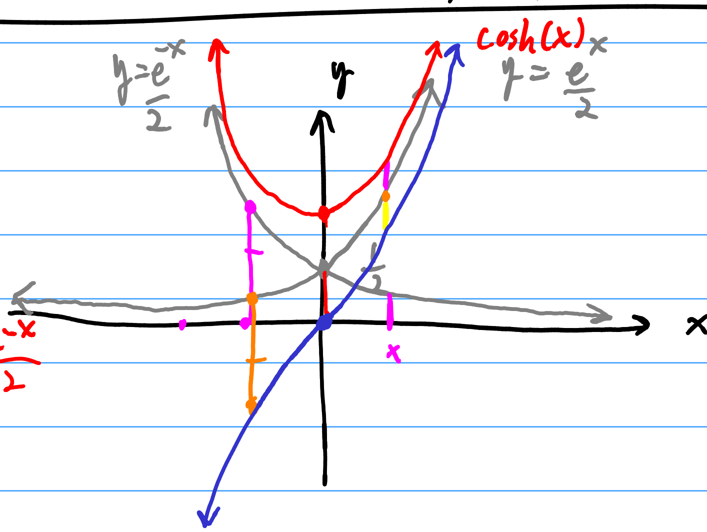
$$\left.\begin{array}{l} 1.\ \cosh(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right) \\ 2.\ \sinh(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right) \end{array}\right\}$$

$$\cosh^2(x) = \left(\frac{1}{2}\left(e^{x} + e^{-x}\right)\right)^2 = \frac{1}{4}\left(e^{x} + e^{-x}\right)\left(e^{x} + e^{-x}\right) = \frac{1}{4}\left(e^{2x} + 2 + e^{-2x}\right)$$

$$-\ \sinh^2(x) = \left(\frac{1}{2}\left(e^{x} - e^{-x}\right)\right)^2 = \frac{1}{4}\left(e^{x} - e^{-x}\right)\left(e^{x} - e^{-x}\right) = -\frac{1}{4}\left(e^{2x} - 2 + e^{-2x}\right)$$

$$\cosh^2(x) - \sinh^2(x) = \frac{1}{4}\left(2 - (-2)\right) = \frac{1}{4} \cdot 4 = 1 \quad \checkmark$$

Graphs!



$\cosh(x) = \frac{e^{x}}{2} + \frac{e^{-x}}{2}$

Domain: $\mathbb{R}$
Range: $[1, \infty)$

$\sinh(x) = \frac{e^{x}}{2} - \frac{e^{-x}}{2}$

Domain: $\mathbb{R}$
Range: $\mathbb{R}$

Recall:

$(e^u)' = e^u \cdot u'$

Ex. $\frac{d}{dx}[\cosh(x)] = \frac{d}{dx}\left[\frac{1}{2}(e^x + e^{-x})\right]$

$= \frac{1}{2}(e^x - e^{-x}) = \sinh(x)$

so, $\boxed{(\cosh(x))' = \sinh(x).}$

Ex. $\frac{d}{dx}[\sinh(x)] = \frac{d}{dx}\left[\frac{1}{2}(e^x - e^{-x})\right]$

$= \frac{1}{2}(e^x + e^{-x}) = \cosh(x)$

$\boxed{(\sinh(x))' = \cosh(x)}$

Ex. $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

$\coth(x) = \frac{\cosh(x)}{\sinh(x)}$

$\operatorname{sech}(x) = \frac{1}{\cosh(x)}$

$\operatorname{csch}(x) = \frac{1}{\sinh(x)}$

$\frac{d}{dx}[\tanh(x)] = \frac{d}{dx}\left[\frac{\sinh(x)}{\cosh(x)}\right] = \frac{\cosh(x)\cdot(\sinh(x))' - \sinh(x)\cdot(\cosh x)'}{\cosh^2(x)}$

HPT

$= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$

$= \frac{1}{\cosh^2 x} = \operatorname{sech}^2(x)$

so, $\boxed{(\tanh(x))' = \operatorname{sech}^2(x)}$

Ex. $\frac{d}{dx}[\text{sech}(x)] = \frac{d}{dx}\left[\frac{1}{\cosh(x)}\right] = \frac{d}{dx}\left[\frac{1}{u}\right] = \frac{-1}{u^2} \cdot u'$

$$= \frac{-1 \cdot \sinh(x)}{\cosh(x)\ \cosh(x)} = \frac{-1}{\cosh(x)} \cdot \frac{\sinh(x)}{\cosh(x)}$$

$$= -\text{sech}(x) \cdot \tanh(x)$$

$$\boxed{(\text{sech}(x))' = -\text{sech}(x)\tanh(x)}$$