Name: Key

M243: Calculus II (Fall 2018)
Midterm Exam, part I

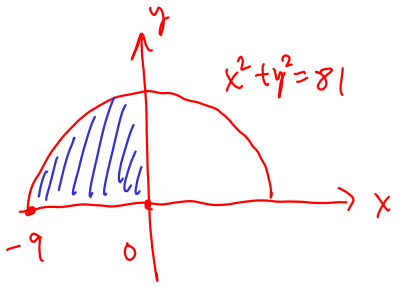
Read and follow all instructions. You may not use any electronic devices, but you may use one 3×5 in^2 index card of your own hand-written notes.

Part I: Computations

Compute the definite and indefinite integrals, showing enough work. You may use any legitimate method. Each problem is worth 10 points. Partial credit will be given when deserved.

Be sure to treat any improper integrals properly.

1. $\displaystyle\int_{-9}^{0} \sqrt{81 - x^2}\, dx$ $= \frac{1}{4}\pi r^2 = \boxed{\frac{81\pi}{4}}$ by geometry



2. $\displaystyle\frac{1}{2}\int \frac{2x}{\sqrt{x^2+4}}\, dx$ $= \frac{1}{2}\int u^{-1/2}\, du = \frac{1}{2}\cdot 2u^{1/2} + C = \boxed{\sqrt{x^2+4} + C}$

$u = x^2+4$

$du = 2x\, dx$

3. $\displaystyle\int \frac{dx}{x^2+6x+10}$ $= \displaystyle\int \frac{1}{(x+3)^2+1}\,dx = \boxed{\arctan(x+3)+C}$

4. $\displaystyle\int \cos^2\theta \sin^2\theta\, d\theta = \int (\cos\theta\sin\theta)^2\,d\theta = \int \left(\tfrac{1}{2}\sin(2\theta)\right)^2 d\theta = \tfrac{1}{4}\int \tfrac{1}{2}\left(1-\cos(4\theta)\right) d\theta$

$= \tfrac{1}{8}\displaystyle\int d\theta - \tfrac{1}{8}\int \cos 4\theta\, d\theta$

$= \boxed{\tfrac{1}{8}\theta - \tfrac{1}{32}\sin(4\theta)+C}$

5. $\displaystyle\int_0^1 \frac{2}{(x+1)(x^2-x-2)}\,dx$ $= 2\displaystyle\int_0^1 \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2}\,dx$

$1 = A(x^2-x-2) + B(x-2) + C(x+1)^2$

$x=-1$: $1 = -3B \Rightarrow B = -1/3$

$x=2$: $1 = 9C \Rightarrow C = 1/9$

$x=0$: $1 = -2A - 2B + C$

$2A = C - 2B - 1 = 1/9 + 2/3 - 1 = -4/9$

$A = -2/9$

$= 2A\ln|x+1| - \dfrac{2B}{x+1} + 2C\ln|x-2|\Big|_0^1$

$= 2A\ln 2 - 2A\ln 1 - \dfrac{2B}{2} + \dfrac{2B}{1} + 2C\ln(1) - 2C\ln(2)$

$= (2A-2C)\ln 2 + B$

$= 2(-2/9 - 1/9)\ln 2 - 1/3$

$= \boxed{-\frac{2}{3}\ln 2 - \frac{1}{3}}$

6. $\displaystyle\int_1^2 \frac{1}{\sqrt{x-1}}\,dx$ $= \lim_{t\to 0^+} \int_t^1 u^{-1/2}\,du = \lim_{t\to 0^+} 2\sqrt{u}\Big|_t^1 = \lim_{t\to 0^+} 2 - 2\sqrt{t} = 2 - 0 = \boxed{2}$

$u = x-1$ $\quad u(2)=1$

$du = dx$ $\quad u(1)=0$