M243 9/7/18

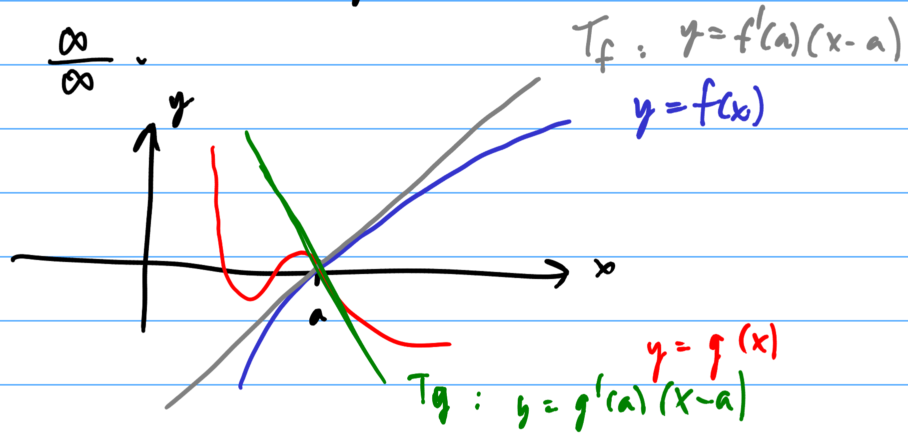
§6.8 L'Hôpital's Rule ☺

Def'n. Suppose f and g are functions such that

$$\lim_{x\to a} f(x)=0 \quad \underline{\text{and}} \quad \lim_{x\to a} g(x)=0,$$

then $$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

This also works for $\frac{\infty}{\infty}$.



$\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, $0\cdot\infty$, $\infty-\infty$ are examples of <u>indeterminate</u> <u>forms</u>.

<u>Ex.</u> $$\lim_{x\to 1}\frac{\ln(x)}{x-1} = \frac{\ln(1)}{1-1} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 1}\frac{1/x}{1} = \lim_{x\to 1}\frac{1}{x}$$

$$=\frac{1}{1}=1$$

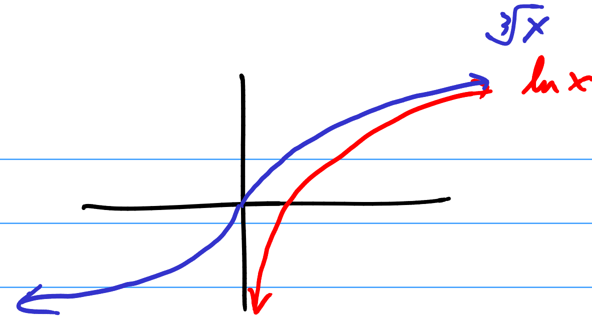
<u>Ex.</u> $$\lim_{x\to\infty}\frac{e^x}{x^2} = \frac{e^\infty}{\infty^2} = \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty}\frac{e^x}{2x} = \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to\infty}\frac{e^x}{2} = \frac{\infty}{2} = \infty$$

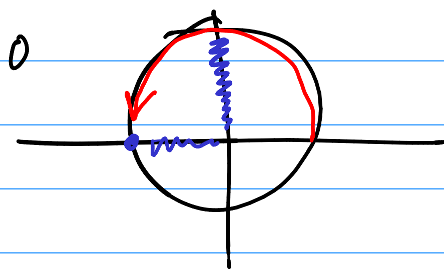
Ex. $\lim\limits_{x\to\infty} \dfrac{\ln x}{\sqrt[3]{x}} = \dfrac{\infty}{\infty}$

$\overset{L'H}{=} \lim\limits_{x\to\infty} \dfrac{1/x}{\frac{1}{3}x^{-2/3}} = \lim\limits_{x\to\infty} \dfrac{3}{\sqrt[3]{x}}$

$= \dfrac{3}{\infty} = 0$



Ex. $\lim\limits_{x\to\pi^-} \dfrac{\sin x}{1-\cos x} = \dfrac{\sin\pi}{1-\cos\pi} = \dfrac{0}{1-(-1)} = \dfrac{0}{2} = 0$



what if?

$\neq \lim\limits_{x\to\pi^-} \dfrac{\cos x}{\sin x} = \dfrac{-1}{0^+} = -\infty$ ← WRONG.

Ex. $\lim\limits_{x\to 0^+} x\ln x = 0\cdot(-\infty)$ Indeterminate

$\parallel$

$\lim\limits_{x\to 0^+} \dfrac{\ln x}{1/x} = \dfrac{-\infty}{+\infty} \overset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{1/x \cdot x^2}{-1/x^2 \cdot x^2} = -\lim\limits_{x\to 0^+} \dfrac{x^2}{x}$

$= -\lim\limits_{x\to 0^+} x = 0$

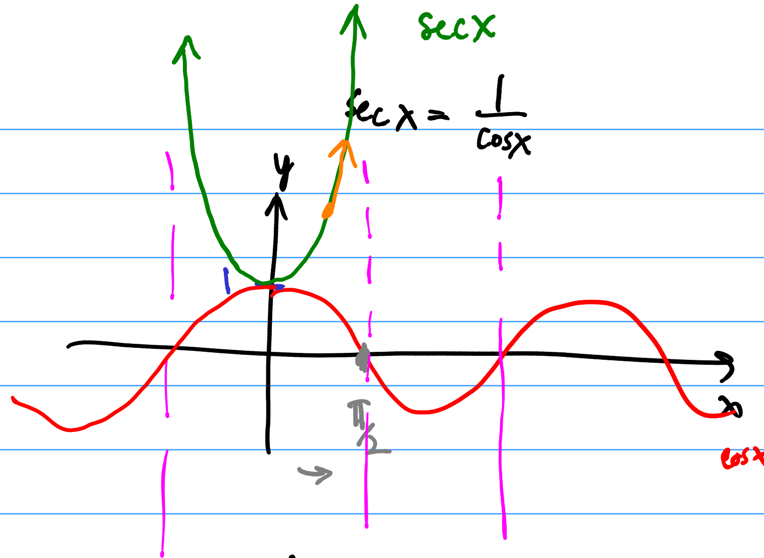
Ex. $\lim\limits_{x\to -\infty} xe^x = -\infty\, e^{-\infty} = \dfrac{-\infty}{e^\infty} = \dfrac{-\infty}{\infty}$

$\parallel \longrightarrow \lim\limits_{x\to -\infty} \dfrac{x}{e^{-x}} \overset{L'H}{=} \lim\limits_{x\to -\infty} \dfrac{1}{-e^{-x}} = \dfrac{1}{-e^{\infty}} = \dfrac{1}{-\infty} = 0.$

OR.

$\parallel \longrightarrow \lim\limits_{x\to -\infty} \dfrac{e^x}{1/x} \overset{L'H}{=} \lim\limits_{x\to -\infty} \dfrac{e^x}{-1/x^2} = \lim\limits_{x\to -\infty} -x^2 e^x$

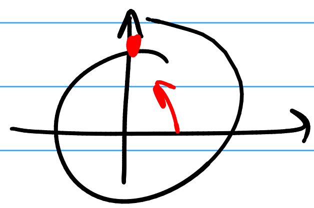
$\underline{Ex}.$ $\displaystyle\lim_{x\to(\pi/2)^-}\left(\sec x-\tan x\right)$

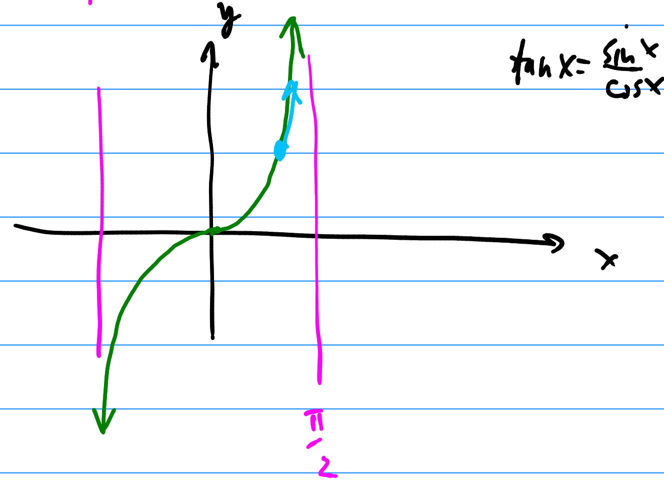


$= \infty - \infty = ?$

$= \displaystyle\lim_{x\to(\pi/2)^-}\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)$

$= \displaystyle\lim_{x\to(\pi/2)^-}\left(\frac{1-\sin x}{\cos x}\right)$





$= \dfrac{1-1}{0} = \dfrac{0}{0}$

$\overset{L'H}{=} \displaystyle\lim_{x\to(\pi/2)^-}\frac{\cancel{-}\cos x}{\cancel{-}\sin x} = \frac{0}{1} = 0$

$\displaystyle\lim_{x\to\frac{\pi}{2}^-} \sec x-\tan x = 0$