M243 9/10/18

Ex. $\frac{d}{dx}\left(\tan^{-1}(5x^2y) = x + 3xy^2\right)$ Find $\frac{dy}{dx}$.

$$\frac{1}{1+(5x^2y)^2}\left(10xy + 5x^2\frac{dy}{dx}\right) = 1 + 3y^2 + 3x\left(2y\frac{dy}{dx}\right)$$

$$10xy + 5x^2\frac{dy}{dx} = \left(1+(5x^2y)^2\right)\left(1+3y^2+6xy\frac{dy}{dx}\right)$$

$$10xy + 5x^2\frac{dy}{dx} = 1 + 3y^2 + 6xy\frac{dy}{dx} + 25x^4y^2 + 75x^4y^4 + 150x^5y^3\,dy/dx$$

$$\frac{dy}{dx}\left(5x^2 - 6xy - 150x^5y^3\right) = 1 + 3y^2 + 25x^4y^2 + 75x^4y^4 - 10xy$$

$$\boxed{\frac{dy}{dx} = \frac{1+3y^2+25x^4y^2+75x^4y^4-10xy}{5x^2-6xy-150x^5y^3}}$$

Ch 7 - Integration

§7.1 - Integration by Parts

$$\int \frac{d}{dx}\left[f(x)\,g(x)\right] = \int \frac{d}{dx}\left[f(x)\right] g(x) + \int f(x)\frac{d}{dx}\left[g(x)\right]$$

$$f(x)g(x) = \int g(x)\frac{d}{dx}\left[f(x)\right] + \underbrace{\int f(x)\frac{d}{dx}\left[g(x)\right]}_{\text{solve for this}}$$

$$\int f(x)\frac{d}{dx}\left[g(x)\right] = f(x)g(x) - \int g(x)\frac{d}{dx}\left[f(x)\right]$$

$u = f(x)$ $\quad du = f'(x)\,dx$

$v = g(x)$ $\quad dv = g'(x)\,dx$

$$(*) \quad \boxed{\int u\,dv = uv - \int v\,du} \qquad \text{IbP}$$

Ex. $\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$

$u = x$ $\quad du = dx$

$dv = \cos x\,dx$ $\quad v = \sin x$

Ex. $\int x^2 e^x\,dx = x^2e^x - 2\int x e^x\,dx = x^2e^x - 2\left(xe^x - \int e^x dx\right)$

$u = x^2$ $\quad dv = e^x dx$; $\quad u = x$ $\quad dv = e^x dx$

$du = 2x\,dx$ $\quad v = e^x$; $\quad du = dx$ $\quad v = e^x$

$$\int x^2e^x dx = x^2e^x - 2xe^x + 2e^x + C$$

Ex. $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2\frac{1}{x}\,dx = \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx$

~~$u = x$ $\quad dv = \ln x\,dx$~~

~~$du = dx$ $\quad v =$~~ ; $\quad u = \ln x$ $\quad dv = x\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{1}{2}x^2$

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

Ex. $\int \ln x\,dx = x\ln x - \int x\frac{1}{x}\,dx = x\ln x - \int 1\,dx$

$u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x}\,dx$ $\quad v = x$

$$\boxed{\int \ln x\,dx = x\ln x - x + C}$$

Q. How do we choose u?

- I - Inverse Trig.
- L - Logarithms
- A - Algebraic (polynomials)
- T - Trig.
- E - Exponential

Ex. $\int e^x \cos x \, dx = e^x \cos x - \int e^x(-\sin x)\,dx = e^x \cos x + \int e^x \sin x \, dx$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x \, dx \qquad v = e^x$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$+\int e^x \cos x \, dx \qquad\qquad +\int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\boxed{\int e^x \cos x \, dx = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C}$$

Ex. $\int x^5 e^x\,dx = x^5e^x - 5x^4e^x + 20x^3e^x - 60x^2e^x + 120xe^x - 120e^x + C$

	u	$dv = \_\_\_\,dx$
$+$	x^5	e^x
$-$	$5x^4$	e^x
$+$	$20x^3$	e^x
$-$	$60x^2$	e^x
$+$	$120x$	e^x
$-$	120	e^x
	0	e^x

Ex. $\int x^3 \sin(2x)\,dx = -\frac{1}{2}x^3\cos(2x) + \frac{3}{4}x^2\sin(2x) + \frac{3}{4}x\cos(2x) - \frac{3}{8}\sin(2x) + C$

	u	$dv = \_\_\_\,dx$
$+$	x^3	$\sin(2x)$
$-$	$3x^2$	$-\frac{1}{2}\cos(2x)$
$+$	$6x$	$-\frac{1}{4}\sin(2x)$
$-$	6	$\frac{1}{8}\cos(2x)$
	0	$\frac{1}{16}\sin(2x)$