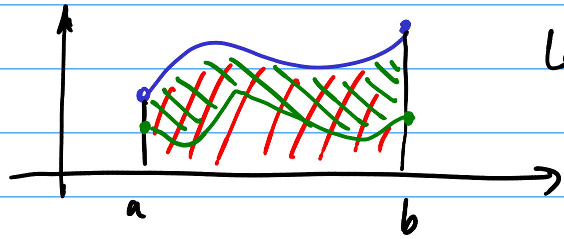
M243 10/2/18

Review:



Lamina has constant density ρ.

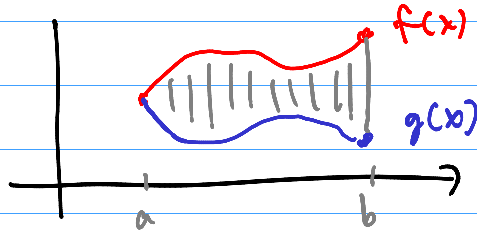
$\rho = 1$.

Moments of mass: $M_y = \int_a^b x f(x)\, dx$

$M_x = \int_a^b \frac{1}{2}[f(x)]^2\, dx$

Center of mass: $(\bar{x}, \bar{y}) = \left(\frac{M_y}{A}, \frac{M_x}{A}\right)$

If the lamina is bounded above and below by functions

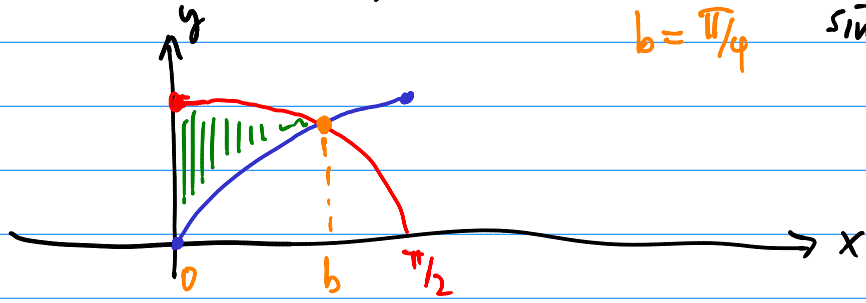


Moments: $M_y = \int_a^b x\,[f(x) - g(x)]\, dx$

$M_x = \int_a^b \frac{1}{2}\left[f(x)^2 - g(x)^2\right] dx$

Ex. Find the centroid (center of mass) of the region bounded by the y-axis, $y = \sin x$, $y = \cos x$.

$b = \pi/4$ since $\sin(\pi/4) = \cos(\pi/4)$



Start w/ area: $A = \int_0^{\pi/4} \cos x - \sin x \, dx$

$$= \sin x + \cos x \Big|_0^{\pi/4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0+1) = \underbrace{\sqrt{2} - 1}_{A}$$

$$M_y = \int_0^{\pi/4} x \left[\cos x - \sin x\right] dx$$

$u = x$ $\qquad dv = (\cos x - \sin x)\,dx$

$du = dx$ $\qquad v = \sin x + \cos x$

$$= x(\sin x + \cos x) - \int \sin x + \cos x \, dx \Big]_0^{\pi/4}$$

$$= x(\sin x + \cos x) + \cos x - \sin x \Big|_0^{\pi/4}$$

$$\sqrt{2}\,\pi/4 + 0 - (0 + 1 - 0) = \frac{\sqrt{2}\pi}{4} - 1 = \frac{\sqrt{2}\pi - 4}{4}$$

$$M_x = \frac{1}{2}\int_0^{\pi/4} \cos^2 x - \sin^2 x \, dx = \frac{1}{2}\int_0^{\pi/4} \frac{1}{2} + \frac{1}{2}\cos(2x) - \left[\frac{1}{2} - \frac{1}{2}\cos(2x)\right] dx$$

$$= \frac{1}{2}\int_0^{\pi/4} \cos(2x)\,dx$$

$$= \frac{1}{4}\sin(2x)\Big|_0^{\pi/4} = \frac{1}{4}$$

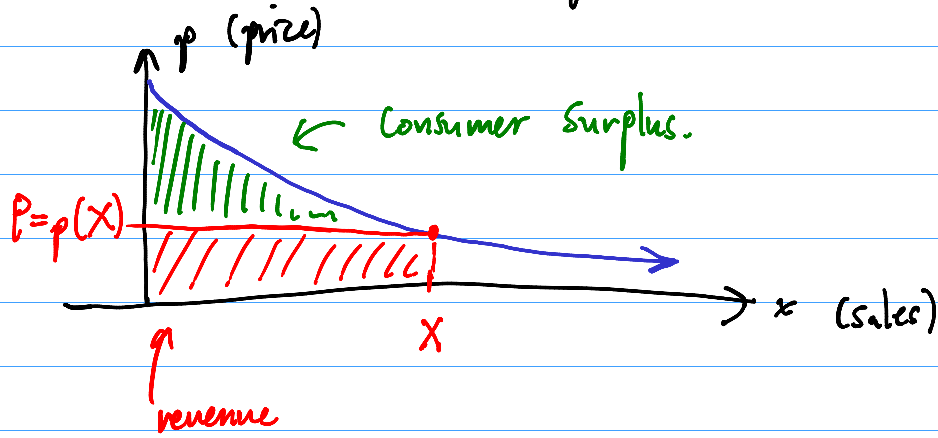
$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{A}, \frac{M_x}{A}\right) = \left(\frac{\sqrt{2}\pi - 4}{4(\sqrt{2}-1)}, \frac{1}{4(\sqrt{2}-1)}\right) \leftarrow \text{correct.}$$

$$\approx (0.267, \underline{0.345})$$

?

§8.4 – Applications to Economics and Biology

Economics:



$$CS = \int_0^X p(x) - P \, dx$$

Ex. $p(x) = 1200 - 0.2x - 0.0001x^2$
Find the consumer surplus when $X=500$.

$$CS = \int_0^{500} 1200 - 0.2x - 0.0001x^2 - \underbrace{p(500)} \, dx$$

$$p(500) = 1200 - \frac{1}{5}(500) - \frac{1}{10000} 250000$$

$$= 1200 \underbrace{- 100 - 25} = 1075$$

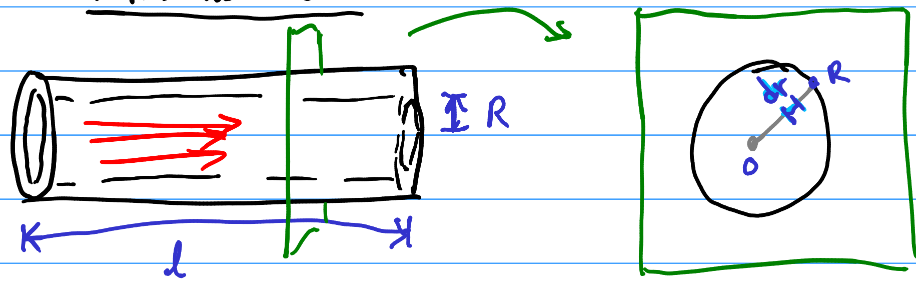
$$CS = \int_0^{500} 125 - 0.2x - 0.0001x^2 \, dx$$

$$= 125x - 0.1x^2 - \frac{0.0001}{3} x^3 \Big|_0^{500}$$

$$= \$33{,}333.33$$

Blood Flow: Derive Poiseuille's Law

Blood vessel:



$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

Total flux, $F = \int_0^R 2\pi r \, v(r)\, dr$

$$= \int_0^R 2\pi r \frac{P}{4\eta l}(R^2 - r^2)\, dr$$

$$= \frac{\pi P}{4\eta l}\int_0^R \underbrace{2r}\,\underbrace{(R^2 - r^2)}\,\underbrace{dr}$$

$u = R^2 - r^2$ $\qquad$ $u(R) = R^2 - R^2 = 0$

$du = -2r\,dr$ $\qquad$ $u(0) = R^2 - 0^2 = R^2$

$$F = \frac{\pi P}{4\eta l}\int_0^{R^2} u\,du = \frac{\pi P}{4\eta l} \cdot \frac{1}{2}u^2 \Big|_0^{R^2}$$

Poiseuille's Law: $\boxed{F = \frac{\pi P R^4}{8\eta l}}$

Ex. Cardiac output : volume of blood pumped per unit of time.

$c = c(t)$ = concentration of dye in blood at time t.
$0 \leq t \leq T$

$$\text{output} = \frac{\text{Amount}}{\int_0^T c(t)\,dt}$$ where we know the total amount of dye injection.

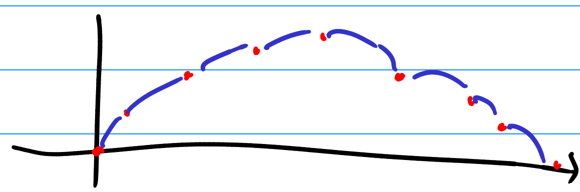
Ex.

t (sec)	$c(t)$
0	0
1	0.4
2	2.8
3	6.5
4	9.8
5	8.9
6	6.1
7	4.0
8	2.3
9	1.1
10	0

5 mg of dye are injected

$$\text{output} = \frac{5}{\underbrace{\int_0^{10} c(t)\,dt}_{\text{Approximate this "somehow"}}}$$

Use Simpson's Rule



$$\int_0^{10} c(t)\,dt \overset{S}{\approx} \frac{1}{3}\left(1(0)+4(0.4)+2(2.8)+\cdots+4(1.1)+1(0)\right)$$

$$= 41.87$$

$$\text{output} = \frac{5}{41.87} = 0.12\ L/s = 7.2\ L/\text{min}$$