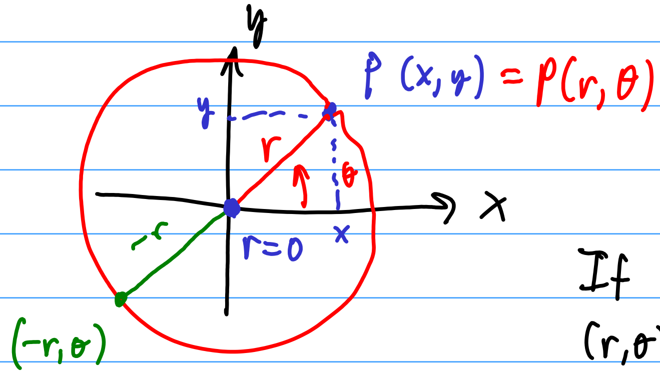
Ch 10 Parametrized Curves

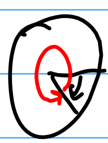
§10.3 Polar Coordinates



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

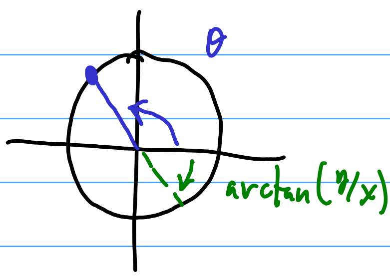
If we know (x,y), then we can determine (r,θ)

if $\underline{0 \leq r}$ and $\underline{0 \leq \theta < 2\pi}$, then we can find unique (r,θ).

IV 

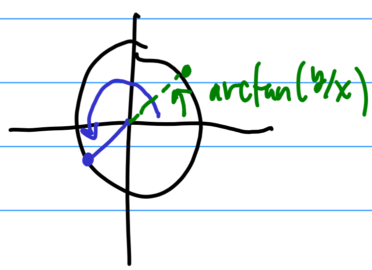
$\rightarrow$ $r = \sqrt{x^2+y^2}$

$\tan\theta = y/x \Rightarrow \theta =$ "$\arctan(y/x)$"



For θ:

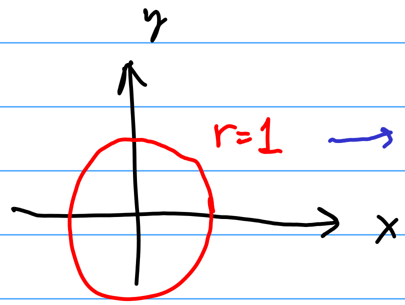
Quadrant	θ
I	$\arctan(y/x)$
II	$\arctan(y/x) + \pi$
III	$\arctan(y/x) + \pi$
IV	$\arctan(y/x) + 2\pi$



Rectangular coordinates are also called Cartesian.

Ex. $r=1$ what does the graph look like?

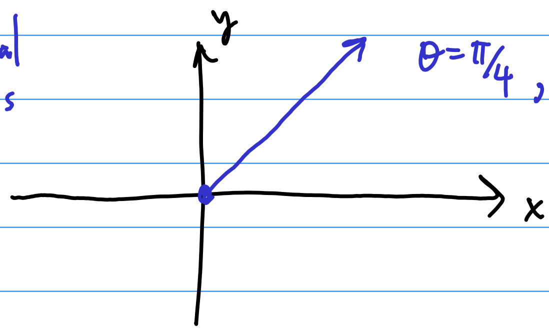
circles

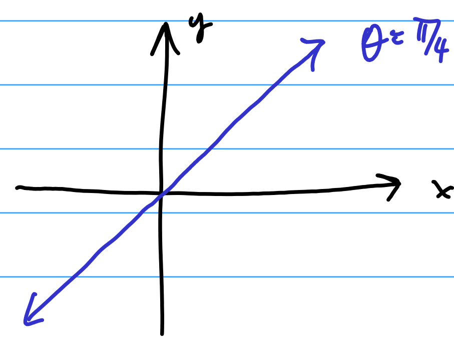


$r=1 \longrightarrow \sqrt{x^2+y^2}=1 \longrightarrow x^2+y^2=1$

Ex. $\theta = \pi/4$, $r \geq 0$ OR if $r \in \mathbb{R}$

radial lines



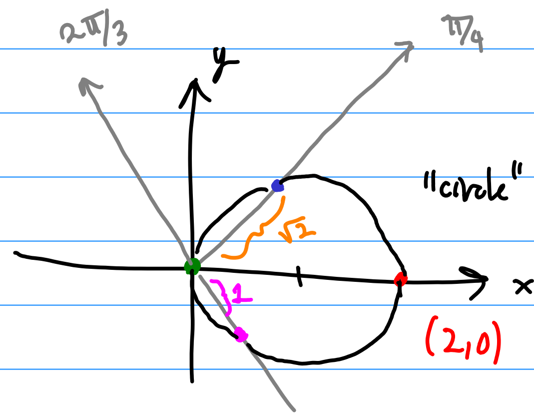


Ex. $r = 2\cos\theta$ consider $r=r(\theta)$ as a function

circle

θ	$r=2\cos\theta$
0	2
$\pi/4$	$2\cdot\frac{\sqrt{2}}{2}=\sqrt{2}$
$\pi/2$	$2\cdot 0=0$
$2\pi/3$	$2\cdot\left(-\frac{1}{2}\right)=-1$
$3\pi/4$	$2\cdot\left(-\frac{\sqrt{2}}{2}\right)=-\sqrt{2}$
π	$2(-1)=-2$

$(-2,\pi)=(2,0)$



Algebraically: $r=2\cos\theta$ but $\cos\theta=\frac{x}{r}$ because $x=r\cos\theta$, so

$$r = 2\frac{x}{r}$$

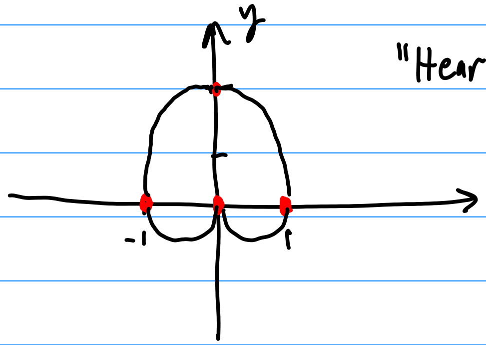
$$r^2 = 2x$$

$$x^2+y^2=2x \longrightarrow x^2-2x+1+y^2=+1$$

$$(x-1)^2+y^2=1^2$$

circle w/ $r=1$, $C=(1,0)$

Ex. **The Cardiod** $r = 1 + \sin\theta$



"Heart?"

$r(0) = 1 + \sin 0 = 1$

$r(\pi/2) = 1 + \sin \pi/2 = 1 + 1 = 2$

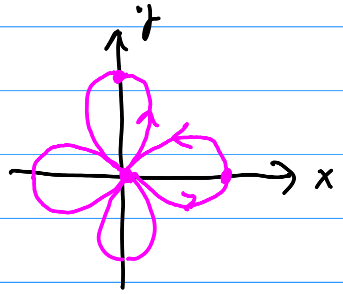
$r(\pi) = 1 + \sin\pi = 1$

$r\left(\frac{3\pi}{2}\right) = 1 + \sin(3\pi/2) = 1 - 1 = 0$

Ex. **Four-leaved Rose** $r = \cos(2\theta)$

$\sin(2\theta) = 2\cos\theta\sin\theta$

$$\cos(2\theta) = \begin{cases} 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \\ \underbrace{\cos^2\theta - \sin^2\theta} \end{cases}$$



$$r = \cos^2\theta - \sin^2\theta$$
$$r = \left(\frac{x}{r}\right)^2 - \left(\frac{y}{r}\right)^2$$
$$r^3 = x^2 - y^2$$
$$(x^2 + y^2)^{3/2} = x^2 - y^2 \leftarrow$$

Idea: if $r = f(\theta)$, then we ~~can~~ may regard any curve as parametrized by θ.

$$\begin{cases} x = r\cos\theta = f(\theta)\cos\theta \\ y = r\sin\theta = f(\theta)\sin\theta \end{cases}$$

then, $$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[f(\theta)\cos\theta\right] = f'(\theta)\cos\theta - f(\theta)\sin\theta$$

... etc...