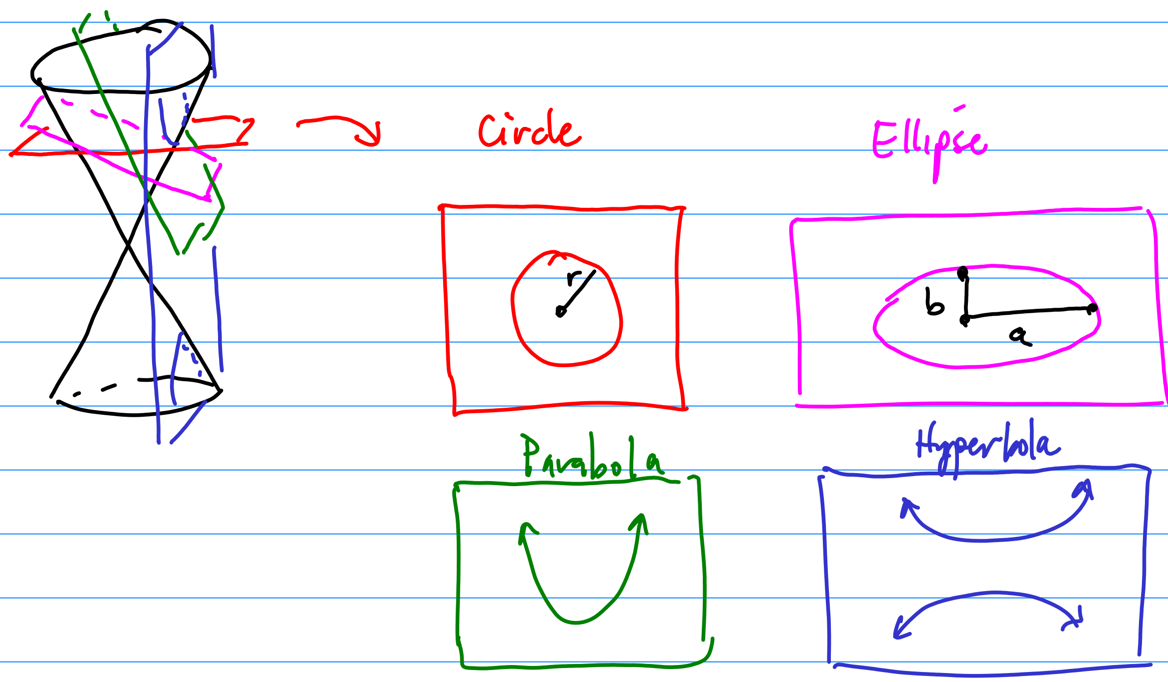
M243 — 24 Oct '18

§10.5, 10.6 – Conic Sections



Circle: $C(h,k)$ w/ radius $r>0$

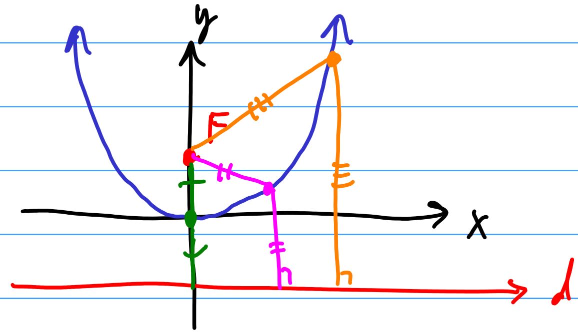
$$(x-h)^2+(y-k)^2=r^2 \quad \text{or} \quad \left(\frac{x-h}{r}\right)^2+\left(\frac{y-k}{r}\right)^2=1$$

Ellipse. $\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1$ $\qquad a,b>0$

Hyperbola: $\left(\frac{x-h}{a}\right)^2-\left(\frac{y-k}{b}\right)^2=1$ $\quad$ or $\quad$ $\left(\frac{y-k}{b}\right)^2-\left(\frac{x-h}{a}\right)^2=1$

Parabola: $y-k=b(x-h)^2$ $\quad$ or $\quad$ $x-h=a(y-k)^2$

Another P.O.V. Parabola



Focus $= F$
directrix $= d$

The parabola is the locus (collection) of points equidistant from F and d.

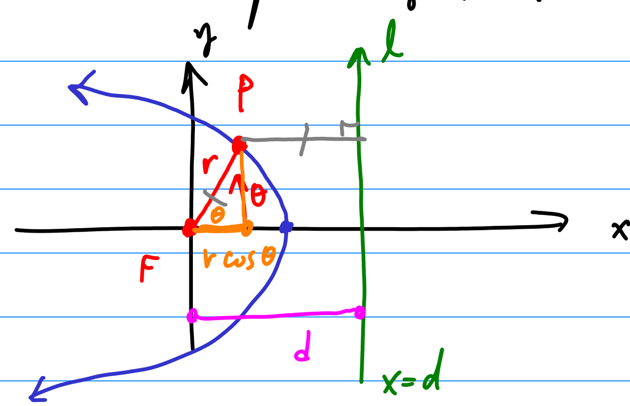
Thm. Let F be a fixed pt in the plane and ℓ be a fixed line in the plane. Let $e > 0$ be a fixed positive number (called the eccentricity). The set of all points in the plane satisfying

$$\rightarrow \boxed{\frac{d(P,F)}{d(P,\ell)} = e}$$

is a conic section.

$e = 1$: Parabola
$e > 1$: Hyperbola
$0 < e < 1$: Ellipse (and circle)

For $e=1$ (parabola) we can derive a polar equation:



$F = (0,0)$, ℓ: $x = d$

$d(P,F) = r$
$d(P,\ell) = d - r\cos\theta$

Then the parabola satisfies:

$$\frac{r}{d - r\cos\theta} = 1$$ Now, solve for r.

$$r = d - r\cos\theta$$
$$r + r\cos\theta = d$$
$$r(1 + \cos\theta) = d$$

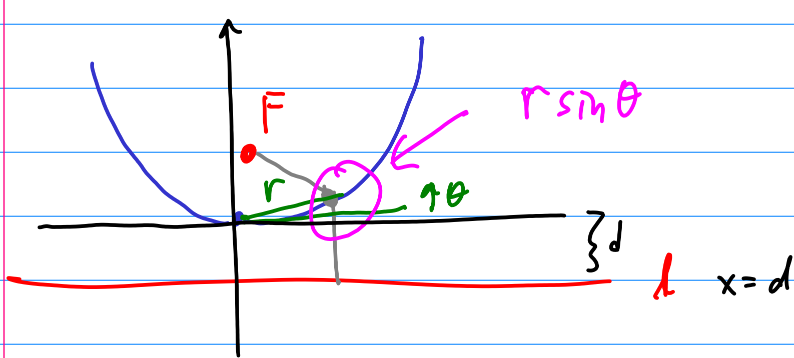
$$\boxed{r(\theta) = \frac{d}{1+\cos\theta}}$$ Polar function!

For the other eccentricities:

$$\frac{r}{d - r\cos\theta} = e \longrightarrow \boxed{r(\theta) = \frac{d\cdot e}{1 + e\cos\theta}}$$

<u>vertical directrix</u>

<u>Horizontal directrix</u>



$$r(\theta) = \frac{e\cdot d}{1 - e\sin\theta}$$

Generic Forms: $r(\theta) = \dfrac{e\cdot d}{1 \pm e\cos\theta}$ or $\dfrac{e\cdot d}{1 \pm e\sin\theta}$

<u>vertical d.</u> <u>horizontal d.</u>

$0 < e \quad < 1 < \infty$

Ex. $r(\theta) = \dfrac{6}{1 - \sin\theta}$

Classify : identify shape, e, d

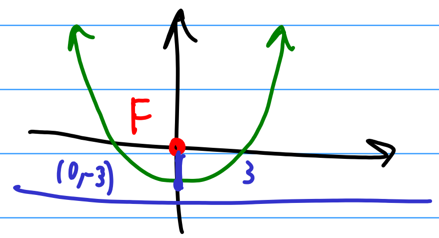
$$r(\theta) = \frac{e \cdot d}{1 \pm \boxed{e} \sin\theta}$$

$e = 1$: parabola

$6 = e \cdot d \rightarrow d = 6$

$\sin\theta$: horizontal

$y = -6$



$y = -6$

$$r(-\pi/2) = \frac{6}{1 - \sin(-\pi/2)} = \frac{6}{1 - (-1)} = \frac{6}{2} = 3$$

Ex. $r = \dfrac{10}{3 - 2\cos\theta}$ $\qquad r(\theta) = \dfrac{e \cdot d}{1 - e\cos\theta}$

$$= \frac{10}{3\left(1 - \frac{2}{3}\cos\theta\right)} = \frac{\boxed{10/3}}{1 - \underbrace{\boxed{2/3}}_{e}\underbrace{\cos\theta}_{x}}$$

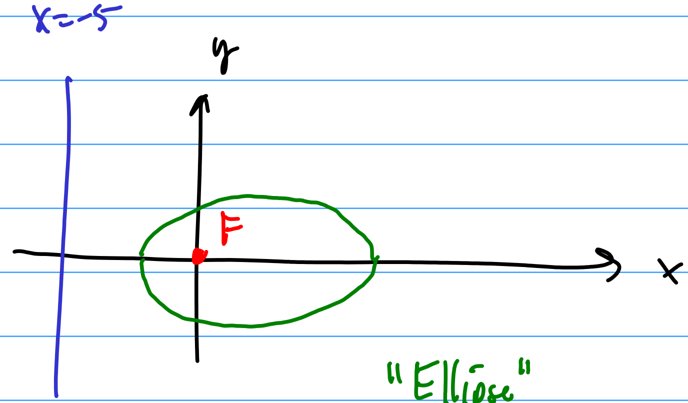
$e = 2/3 < 1$ Ellipse

$e \cdot d = 10/3$

$\frac{2}{3} \cdot d = 10/3$

$d = 5$

$\boxed{x = -5}$



"Ellipse"