M243 2 Nov '18

Ch9 - Differential Equations - Review Exercises.

1. Sketch the slope field for the DE at a collection of points. Sketch an approximate solution curve for initial condition $y(1)=1$. What is the equilibrium solution $(y'=0)$? Solve the DE analytically.

$$\frac{dy}{dx} = 2y - 4$$

2. Find the general solutions of the Second Order DEs:

a.) $2y'' + y' - y = 0$ b.) $y'' + 4y' + 5y = 0$ c.) $y'' + 2y' + y = 0$

3. Solve the initial value problems (IVP):

a.) $\begin{cases} y'' - 6y' - 7y = 0 \\ y(0) = 1 \\ y'(0) = 2 \end{cases}$ b.) $\begin{cases} y'' + 4y = 0 \\ y(0) = 0 \\ y'(0) = 2 \end{cases}$ c.) $\begin{cases} y'' - 8y' + 16y = 0 \\ y(0) = 4 \\ y'(0) = 0 \end{cases}$

Brief Solutions

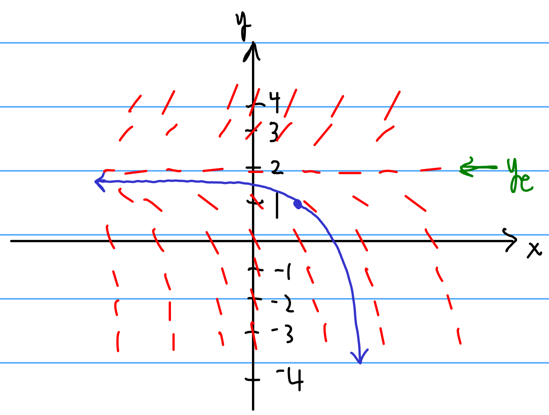
1. Sketch the slope field for the DE at a collection of points. Sketch an approximate solution curve for initial condition $y(1)=1$. What is the equilibrium solution $(y'=0)$? Solve the DE analytically.

$$\frac{dy}{dx} = 2y - 4$$

Equilibrium satisfies $dy/dx = 0$,

so $2y - 4 = 0$

$\Rightarrow y_e = 2$



y	dy/dx
4	4
3	2
2	0
1	-2
0	-4
-1	-6
-2	-8
-3	-8
-4	-10

Analytically: $\frac{dy}{dx} = 2(y-2) \Rightarrow \int \frac{1}{y-2}\, dy = \int 2\, dx \Rightarrow \ln|y-2| = 2x + C$

$y - 2 = Ce^{2x} \Rightarrow \boxed{y = 2 + Ce^{2x}}$

2. a.) $2y''+y'-y=0$

$2r^2+r-1=0$

$2r^2+2r-r-1=0$

$2r(r+1)-1(r+1)=0$

$2(r-\frac{1}{2})(r+1)=0$

$r=\frac{1}{2},-1$

so,

$$\boxed{y=C_1e^{\frac{1}{2}x}+C_2e^{-x}}$$

b.) $y''+4y'+5y=0$

$r^2+4r+5=0$

$r^2+4r+4=-5+4$

$(r+2)^2=-1$

$r=-2\pm 1i$

so,

$$\boxed{y=C_1e^{-2x}\cos x+C_2e^{-2x}\sin x}$$

c.) $y''+2y'+y=0$

$r^2+2r+1=0$

$(r+1)^2=0$

$r=-1,-1$

so,

$$\boxed{y=C_1e^{-x}+C_2xe^{-x}}$$

3. a.) $\begin{cases} y''-6y'-7y=0 \\ y(0)=1 \\ y'(0)=2 \end{cases}$

$r^2-6r-7=0$

$(r-7)(r+1)=0$

$r=-1,7$

$y(x)=C_1e^{-x}+C_2e^{7x}$

$y'(x)=-C_1e^{-x}+7C_2e^{7x}$

$y(0)=C_1+C_2=1$

$y'(0)=-C_1+7C_2=2$

$A=\begin{pmatrix}1&1\\-1&7\end{pmatrix}$ $\bar{b}=\begin{pmatrix}1\\2\end{pmatrix}$

$A_1=\begin{pmatrix}1&1\\2&7\end{pmatrix}$ $A_2=\begin{pmatrix}1&1\\-1&2\end{pmatrix}$

$|A|=8$, $|A_1|=5$, $|A_2|=3$

$C_1=5/8$, $C_2=3/8$

and,

$$\boxed{y(x)=\frac{5}{8}e^{-x}+\frac{3}{8}e^{7x}}$$

b.) $\begin{cases} y''+4y=0 \\ y(0)=0 \\ y'(0)=2 \end{cases}$

$r^2+4=0$

$r=\pm 2i=0\pm 2i$

$y(x)=C_1\cos(2x)+C_2\sin(2x)$

$y'(x)=-2C_1\sin(2x)+2C_2\cos(2x)$

$y(0)=C_1=0$

$y'(0)=2C_2=2 \Rightarrow C_2=1$

So,

$$\boxed{y(x)=\sin(2x)}$$

c.) $\begin{cases} y''-8y'+16y=0 \\ y(0)=4 \\ y'(0)=0 \end{cases}$

$r^2-8r+16=0$

$(r-4)^2=0$

$r=4,4$

$y(x)=C_1e^{4x}+C_2xe^{4x}$

$y'(x)=4C_1e^{4x}+C_2e^{4x}+4C_2xe^{4x}$

$y(0)=C_1+C_2=4$

$y'(0)=4C_1+C_2=0$

$A=\begin{pmatrix}1&1\\4&1\end{pmatrix}$ $\bar{b}=\begin{pmatrix}4\\0\end{pmatrix}$

$A_1=\begin{pmatrix}4&1\\0&1\end{pmatrix}$ $A_2=\begin{pmatrix}1&4\\4&0\end{pmatrix}$

$|A|=-3$, $|A_1|=4$, $|A_2|=-16$

$C_1=-4/3$, $C_2=\frac{16}{3}$

so,

$$\boxed{y(x)=\frac{-4}{3}e^{4x}+\frac{16}{3}xe^{4x}}$$