M243 5 Nov '18

A **Bernoulli differential equation** (named after James Bernoulli) is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Observe that, if $n = 0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

Use the substitution $u = y^{1-n}$ to solve the differential equation.

$$y' + \frac{2}{x}y = 3\frac{y^3}{x^2}$$

$n=3.$

$$u^{\frac{1}{1-n}} = \left(y^{1-n}\right)^{\frac{1}{1-n}}$$

$$\cancel{y = u^{\frac{1}{1-n}} \leftarrow y = u^{-1/2} = \frac{1}{\pm\sqrt{u}}}$$

So, the DE is

$$\frac{du}{dx} + (1-3)\frac{2}{x}u = (1-3)\frac{3}{x^2}$$

$$\frac{du}{dx} \underbrace{-\frac{4}{x}}_{p} u = \underbrace{-\frac{6}{x^2}}_{q}$$

Linear!

$$\mu = e^{\int p\,dx} = e^{\int -\frac{4}{x}dx} = e^{-4\ln x}$$

$$\mu = e^{\ln x^{-4}} = x^{-4}$$

$$u(x) = \frac{1}{\mu}\int \mu q\,dx + \frac{c}{\mu}$$

$$= x^4\int \frac{1}{x^4}\cdot\frac{-6}{x^2}\,dx + Cx^4 = x^4\int -6x^{-6}\,dx + Cx^4$$

$$= \frac{+6}{5}x^4 x^{-5} + Cx^4$$

$$y^{-2} = u = \frac{6}{5}\frac{1}{x} + Cx^4$$

$$\boxed{y(x) = \frac{1}{\pm\sqrt{\frac{6}{5}\frac{1}{x} + Cx^4}}}$$

10. Question Details $v(0)=0$ SCalc8 9.5.035. [3943348]

An object with mass m is dropped from rest and we assume that the air resistance is proportional to the speed of the object. If $s(t)$ is the distance dropped after t seconds, then the speed is $v = s'(t)$ and the acceleration is $a = v'(t)$. If g is the acceleration due to gravity, then the downward force on the object is $mg - cv$, where c is a positive constant, and Newton's Second Law gives

$$m\frac{dv}{dt} = mg - cv.$$

(a) Solve this as a linear equation. (Use v for $v(t)$.)

(b) What is the limiting velocity?

$\lim_{t\to\infty} v(t) =$

(c) Find the distance the object has fallen after t seconds. (Use s for $s(t)$.)

$v(t)$ = unknown function to find.

m, g, c = constants

$$m\frac{dv}{dt} + cv = mg$$

$$\frac{dv}{dt} + \underbrace{\frac{c}{m}}_{p(t)} v = \underbrace{g}_{g(t)}$$

a). $v = \frac{1}{\mu}\left[\int \mu g\, dt + \frac{k}{\mu}\right]$

$$\mu(t) = e^{\int p\, dt} = e^{\int \frac{c}{m} dt} = e^{\frac{c}{m}t}$$

so, $v(t) = e^{-\frac{c}{m}t}\int e^{\frac{c}{m}t}\cdot g\, dt + k e^{-\frac{c}{m}t}$

$$= \cancel{e^{-\frac{c}{m}t}} \cdot g \cdot \frac{m}{c}\cancel{e^{\frac{c}{m}t}} + k e^{-\frac{c}{m}t}$$

$$v = \frac{mg}{c} + k e^{-\frac{c}{m}t}$$

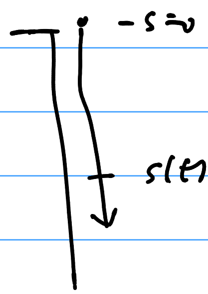
$$0 = v(0) = \frac{mg}{c} + k e^{0} \Rightarrow k = \underbrace{-\frac{mg}{c}}$$

a.) $\boxed{v(t) = \frac{mg}{c} - \frac{mg}{c} e^{-\frac{c}{m}t}}$

b.) $v_\infty = \lim_{t\to\infty}\left(\frac{mg}{c} - \cancel{\frac{mg}{c} e^{-\frac{c}{m}t}}^{\,0}\right) = \boxed{\frac{mg}{c}}$

c.) Find $s(t)$. Recall $v = \frac{ds}{dt}$

$$\int \frac{ds}{\cancel{dt}} = \int\left(\frac{mg}{c} - \frac{mg}{c} e^{-\frac{c}{m}t}\right)dt$$



$$s(t) = \frac{mg}{c}t + \frac{m^2 g}{c^2} e^{-\frac{c}{m}t} + \underbrace{K_2}$$

$s(0) = 0$.

$$\boxed{s(t) = \frac{mg}{c}t + \frac{m^2 g}{c^2} e^{-\frac{c}{m}t} - \frac{m^2 g}{c^2}}$$

Ch 11 ~ Series

§11.1 – Sequences

Def'n: A sequence is a function $a = a(n)$ whose domain is
$\mathbb{N} = 0, 1, 2, 3, \ldots$
and the range is $\mathbb{R}$.

OR

Def'n. A sequence is an ordered list of numbers.

$\{a_n\} = a_0, a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$

or

$\{a_n\}_{n=0}^{\infty}$

Ex. $\left\{ \frac{n}{n+1} \right\}_{n=1}^{\infty} = \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots, \frac{n}{n+1}, \ldots \right\}$

$a_n = \frac{n}{n+1}$ is the general term. This is the function notation also!

Ex. $\left\{ \sqrt{n-3} \right\}_{n=3}^{\infty} = \left\{ \sqrt{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \ldots, \sqrt{n-3}, \ldots \right\}$

$= \{ 0, 1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \ldots \}$

$= \left\{ \sqrt{n} \right\}_{n=0}^{\infty}$

Ex. Find a general term:

a.) $\left\{ \frac{-1}{4}, \frac{2}{9}, \frac{-3}{16}, \frac{4}{25}, \ldots \right\}$ $\quad a_n = ?$ $\quad \{a_n\} = \left\{ (-1)^n \frac{n}{(n+1)^2} \right\}_{n=1}^{\infty}$

b.) $\{2, 7, 12, 17, 22, 27, \ldots\}$

$\to a_n = 5n - 3 \quad n = 1, \ldots, \infty$

$a_n = 2 + 5n \quad n = 0, \ldots, \infty$

$\to a_n = 2 + 5(n-1) \quad n = 1, 2, \ldots$

c) $\{5, 1, 5, 1, 5, 1, 5, 1, \ldots\}$

$$\boxed{a_n = \begin{cases} 5 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases} \quad n = 1, \ldots}$$

$$\boxed{a_n = 3 + (-1)^{n+1} \cdot 2} \qquad n = 1, 2, \ldots$$

Ex. Fibonacci Sequence

$a_0 = 1$

$a_1 = 1$

$a_2 = 2$

$a_3 = 3$

$a_4 = 5$

$a_5 = 8$

$\vdots$

$a_n = a_{n-1} + a_{n-2} \leftarrow$ recursion relation

$F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$

$n = 0, 1, 2, \ldots$

F_n is defined recursively.

RG Q. Can we find a formula for F_n that does not depend on the previous F's?