M243 6 Nov'18

10. Question Details SCalc8 9.3.033. [3349190]

An **integral equation** is an equation that contains an unknown function $y(x)$ and an integral that involves $y(x)$. Solve the given integral equation. [*Hint:* Use an initial condition obtained from the integral equation.]

$$y(x) = 2 + \int_4^x [t - ty(t)]\, dt$$

$$y(4) = 2 + \int_4^4 (t - ty)\,dt = 2 + 0 = 2$$

$y(4) = 2$ is the I.C. ←

$$\frac{d}{dx}\left[y(x) = 2 + \int_4^x t - ty \, dt \right]$$

$$\frac{dy}{dx} = \frac{d}{dx}\int_4^x t - ty \, dt$$

$\dfrac{dy}{dx} = x - xy$ is the corresponding DE.

$\dfrac{dy}{dx} = x(1-y)$ Separable!

$$\int \frac{dy}{1-y} = \int x\,dx$$

$$-\ln|1-y| = \frac{1}{2}x^2 + C$$

I.C.:

$$-\cancel{\ln|1-2|}_{0} = \frac{1}{2}4^2 + C$$

$$C = -\frac{1}{2}4^2 = -8$$

$$-\ln|1-y| = \frac{1}{2}x^2 - 8$$

$$e^{\ln|1-y|} = e^{8 - \frac{1}{2}x^2}$$

$$1 - y = e^8 e^{-\frac{1}{2}x^2}$$

$$\boxed{y(x) = 1 - e^8 e^{-\frac{1}{2}x^2}}$$

§11.1 Sequences

Let $\{a_n\} = \{a_0, a_1, a_2, a_3, \cdots, a_n, \cdots\}$

Q. Can we take a limit of the sequence? Yes!

Def'n. A sequence $\{a_n\}$ has limit L, and we write

$$\lim_{n\to\infty} a_n = \lim a_n = L,$$

if $|a_n - L| < \varepsilon$ whenever $n > N(\varepsilon)$.

Our job is to find $N = N(\varepsilon)$.

Ex. $a_n = \frac{n}{2n+1}$ $\qquad \lim a_n = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$

For $\varepsilon > 0$, let $N(\varepsilon) = \frac{1}{4\varepsilon} - \frac{1}{2}$. Then when $n > N$,

$$|a_n - L| = \left|\frac{n}{2n+1} - \frac{1}{2}\right| = \left|\frac{2n-(2n+1)}{2(2n+1)}\right| = \left|\frac{1}{2(2n+1)}\right| < \varepsilon$$

?

$\frac{1}{2(2n+1)} < \varepsilon$ solve for n.

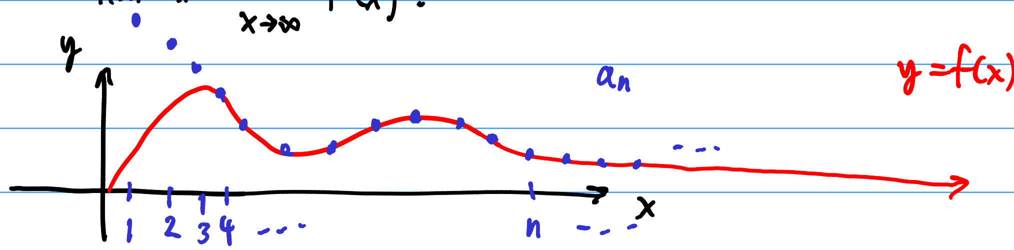
$$2(2n+1) > \frac{1}{\varepsilon} \Rightarrow 2n+1 > \frac{1}{2\varepsilon}$$

$$\Rightarrow 2n > \frac{1}{2\varepsilon} - 1$$

$$n > \frac{1}{4\varepsilon} - \frac{1}{2}$$

Thm. If f is a real-valued function such that $f(n) = a_n$ for all $n > N$, then

$$\lim a_n = \lim_{x\to\infty} f(x).$$



Ex. $\lim \frac{1}{n^r}$ for $r>0$

$$\lim_{n\to\infty} \frac{1}{n^r} = 0$$

Def'n. If $\lim a_n = \text{DNE}$, then we say $\{a_n\}$ diverges.
If $\lim a_n = L \neq \infty$, then $\{a_n\}$ converges.

Squeeze Thm Applies!

If $b_n \leq a_n \leq c_n$ for all $n \geq N$, and if $\lim b_n = \lim c_n = L$, then $\lim a_n = L$.

Ex. If $\lim |a_n| = 0$, then $\lim a_n = 0$.

since $-a_n \leq a_n \leq a_n$ or $0 \leq |a_n|$

Ex. $\lim \frac{(-1)^n}{n}$ $\left\{\frac{(-1)^n}{n}\right\} = \left\{\frac{-1}{1}, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \frac{-1}{5}, \frac{1}{6}, \dots\right\}$

$0! = 1$ $\lim \left|\frac{(-1)^n}{n}\right| = \lim \frac{1}{n} = 0$, so $\lim \frac{(-1)^n}{n} = 0$ also.

Ex. $\lim \frac{n!}{n^n} = \lim \frac{n\cdot(n-1)\cdot(n-2)\cdots 3\cdot 2\cdot \boxed{1}}{n\cdot n\cdot n \cdots n\cdot n\cdot \boxed{n}} = \lim \frac{1}{n}\underbrace{\left(\frac{2\cdot 3\cdot 4\cdots n}{n\cdot n\cdot n\cdots n}\right)}_{\leq 1}$

$$0 \le \frac{n!}{n^n} \le \frac{1}{n}$$

Squeeze! $\lim 0 = 0$ $\lim \frac{1}{n} = 0$

then, $\lim \frac{n!}{n^n} = 0$.

Ex. $a_n = r^n$ for $r > 0$

Ex. $a_n = \frac{1}{2}^n = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots, \frac{1}{2^n}, \dots\}$

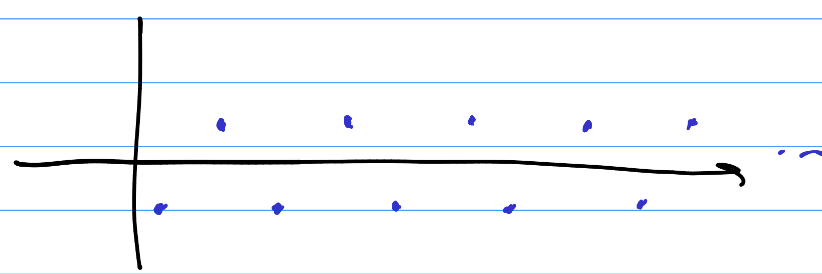
$b_n = 2^n = \{2, 4, 8, 16, 32, \dots\}$

$$\lim r^n = \begin{cases} 0 & \text{if } 0 \le |r| < 1 \\ \infty & \text{if } r > 1 \end{cases}$$

and $\lim 1^n = 1$

$\lim (-1)^n =$ DNE

$\{(-1)^n\} = \{-1, 1, -1, 1, -1, 1, \dots\}$



Ex. $a_n = (1 + \frac{2}{n})^n$ $\lim (1 + \frac{2}{n})^n = ?$

$\{a_n\} = \{(1+2)^1, (1+\frac{2}{2})^2, (1+\frac{2}{3})^3, (1+\frac{1}{2})^4, (\underbrace{1+\frac{2}{5}}_{1})^{5 \leftarrow \infty}, \dots$

$= \{3, 4,$

Replace $a_n = (1+\frac{2}{n})^n$ by $f(x) = (1+\frac{2}{x})^x$

$$\lim a_n = \underbrace{\lim_{x\to\infty} (1+\tfrac{2}{x})^{x} = y}$$

$$\ln y = \ln\left(\lim_{x\to\infty} (1+\tfrac{2}{x})^x\right)$$

$$\ln y = \lim_{x\to\infty} x \ln(1+\tfrac{2}{x}) = \infty \cdot \ln 1 = \infty \cdot 0$$

$$= \lim_{x\to\infty} \frac{\ln(1+2/x)}{1/x} = \frac{0}{0} \quad \text{L'Hôpital}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+2/x} \cdot \frac{-2}{\cancel{x^2}}}{-1/\cancel{x^2}} = \lim_{x\to\infty} \frac{2}{1+2/x} = \frac{2}{1} = 2$$

$$\ln y = 2$$
$$y = e^2$$

$$\boxed{\lim (1+\tfrac{2}{n})^n = e^2}$$

in general, $\lim (1+\frac{k}{n})^n = e^k$